UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Physics 1B27: Introductory Classical Mechanics

COURSE CODE	:	**PHYS1B27**
UNIT VALUE	:	**0.50**
DATE	:	**14-MAY-04**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

04-C1073-3-40

TURN OVER

Attempt all six questions from section A and three questions from section B.

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Acceleration due to gravity at the Earth's surface, g = 9.8 $\mathrm{ms^{-2}}$.
Vector quantities are represented in bold italic symbols, for example $\boldsymbol{F}$.

Where appropriate you may use the following Lorentz transformation equations relating the coordinates of an event as observed in two inertial frames of reference moving relative to each other along the x-axis with velocity v.

$$x' = \gamma(x - vt); y' = y; z' = z; t' = \gamma(t - vx/c^2), \text{where } \gamma = \frac{1}{\sqrt{(1 - v^2/c^2)}}$$

Section A

[Part marks]

1. State the theorems of: *a*) parallel axes; *b*) perpendicular axes, and give the conditions under which each theorem is valid. [4]

   Given that the moment of inertia of a uniform rod of mass M and length L about an axis through one end and perpendicular to the length of the rod is $\frac{1}{3}ML^2$, determine the moments of inertia of a uniform square plate of mass M and sidelength L about the following axes:

   (a) one side of the square;

   (b) through one corner and perpendicular to the plane of the square. [3]

2. State the equation of motion of a particle of mass m attached to a light spring with stiffness constant k that is also acted on by a damping force proportional to the velocity of the particle, the constant of proportionality being λ. [3]

   For critical damping the general solution to the equation of motion of the particle is

   $$x = (A + Bt)\exp\left[\frac{-\lambda t}{2m}\right],$$

   where A and B are arbitrary constants. If the particle is released from rest with a displacement x_o at time $t = 0$ determine the values of A and B and sketch the subsequent displacement x as a function of time. [4]

3. State the infinitesimal amount of work done by a force $\boldsymbol{F}$ when its point of application moves a distance $d\boldsymbol{r}$. Define power P and prove that when the point of application of a force $\boldsymbol{F}$ is moving at a velocity $\boldsymbol{v}$ the power expended is $P = \boldsymbol{F}.\boldsymbol{v}$. **[3]**

   Determine the power required to pull a body of mass m up a smooth plane inclined at an angle α to the horizontal at a constant speed v_o. Explain why it is unnecessary to specify the direction in which the applied force is acting. **[3]**

4. Define the potential energy function $V(\boldsymbol{r})$ for a particle which is acted on by a conservative force $\boldsymbol{F}(\boldsymbol{r})$, given that $V = 0$ at $\boldsymbol{r} = \boldsymbol{r_o}$. Also, explain how $\boldsymbol{F}$ can be determined if V is known. **[3]**

   A particle moving in the $x - y$ plane is subject to a conservative force $\boldsymbol{F}(x, y)$ whose potential function is $V = Kx^3y^2$, where K is a constant.

   Evaluate $\boldsymbol{F}(x, y)$. Also, determine the work done on the particle by this force in moving it from the origin, $x = 0$, $y= 0$, to the point $x = 2$, $y= 4$. **[4]**

5. The centre of mass of the combined system of a person sitting on a bicycle is a distance h above the ground when the bicycle is upright. However, in order to ride at a constant speed v around a circular horizontal rough track of radius R (to the centre of mass of the combined system) the bicycle must be tilted inwards at an angle α to the vertical. Sketch the system as viewed from the front and indicate where the real forces act, and determine their magnitudes. **[3]** Also, determine the magnitude, direction and point of application of the fictitious force which, when introduced, allows you to consider the system as being in static equilibrium in its own frame of reference. Take moments about any convenient point to determine α in terms of R, v and g. **[3]**

6. The muon is an unstable particle with a mean life in its own rest frame of approximately 2×10^{-6}s so that, even travelling at the speed of light, it would be expected, according to nonrelativistic mechanics, to travel only approximately 600 m before decaying. However, muons created as a result of the interactions of cosmic rays with nuclei in the upper atmosphere can be detected on the Earth's surface, several kilometres below. Explain this phenomenon from the viewpoint of:

   (a) an observer on Earth; **[4]**

   (b) an observer at rest relative to the muon. **[3]**

Section B

7. Prove by differentiating the position vector $\boldsymbol{r} = r\hat{\boldsymbol{r}}$ of a particle with respect to the time t that its velocity and acceleration when moving in a plane may be expressed in terms of radial and transverse components as:

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}, \qquad [3]$$

$$\boldsymbol{a} = \ddot{\boldsymbol{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{\boldsymbol{r}} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\boldsymbol{\theta}}, \qquad [5]$$

where $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are unit vectors in the radial and transverse directions respectively.

A particle of mass M is attached to one end of a light inextensible string. The other end of the string is passed through a very small hole in a smooth horizontal table and is held fixed while the particle moves on the table in a circular path of radius r_o with an angular velocity ω_o. At time $t = 0$ the string starts to be pulled through the hole at a constant speed V, so that the distance of the particle from the hole decreases with time according to $r = r_o - Vt$.

State what kinematic property of the system is conserved during the subsequent motion. [2]

Use this information to determine the time variation of the angular velocity of the particle ω. [3]

Hence, determine the time variation of the velocity of the particle $\boldsymbol{v}$ and the tension in the string. [4]

Determine, also, the kinetic energy of the particle. [3]

8. *a*) A rigid body of mass M hangs freely under gravity, supported by a horizontal pivot through a point A on the body. The perpendicular distance from the pivot to C, the centre of mass of the body, is d and the moment of inertia of the body about the pivot is I_A. The body is displaced from its equilibrium position and is then released.

Write down the gravitational torque about the pivot in terms of the angular displacement θ that the line AC makes with the vertical and derive the rotational equation of motion for the body. If the initial angular displacement of the body is small, show that the subsequent motion is simple harmonic and determine the period of oscillation. **[7]**

b) A thin straight rod of length L and mass M has a mass per unit length ρ that increases linearly with the distance x from one end of the rod labelled A, so that $\rho = bx$. Show that the constant $b = 2M/L^2$. **[3]**

Determine the following properties of the rod:

i) the distance of the centre of mass of the rod from A; **[3]**

ii) the moment of inertia of the rod about an axis through A which is perpendicular to the length of the rod; **[4]**

iii) the period for small angle oscillations about a horizontal axis through the end A and perpendicular to the length of the rod (using the expression for the period derived in part *a*). **[3]**

9. Show that a force that is not acting through the centre of mass of a rigid body is equivalent to the force acting through the centre of mass plus a couple that tends to rotate the body about an axis through the centre of mass. **[5]**

A ball of mass m and radius R is set spinning with angular velocity ω_o about a horizontal axis through its centre and is then laid gently on to a rough horizontal plane, the coefficient of friction between the ball and the plane being μ. Initially the centre of mass of the ball is at rest but the frictional force between the ball and the plane causes the ball's centre of mass to accelerate. Furthermore, the torque of this force about the centre of mass causes the angular velocity of the ball to decelerate. Determine the magnitude of the frictional force and indicate on a sketch its direction and point of application. Write down the equations of motion for the centre of mass motion and for the rotational motion, given that the moment of inertia of the ball about an axis through its centre is I_o. Hence, show that the velocity of the centre of mass varies as $v = \mu g t$ and that the angular velocity varies as

$$\omega = \omega_o - \frac{\mu m g R t}{I_o}.$$ **[9]**

State the relationship between the linear velocity v and the angular velocity ω when the ball rolls without skidding, and determine the time at which this first occurs. Also, determine the speed of the centre of mass at this time and the total kinetic energy of the ball. **[6]**

10. A particle of mass M is thrown at a speed u up into the air from ground level in a direction making an angle α with the horizontal, and the particle thereafter moves in a ballistic trajectory under gravity. State the equation of motion of the particle and solve it to obtain the horizontal and vertical components of the particle's velocity and position at a time t after launch. Hence determine the range of the particle on horizontal ground and the time taken to reach the range. **[10]**

A second particle is fired on the same initial trajectory, but at the highest point it explodes into two fragments of equal mass, one of which then falls vertically *from rest* to the ground. Determine the velocity of the other particle immediately after the explosion. Also, determine how far from the original launch point the two particles land. **[10]**

11. *a*) State the relationship between the magnitude of the linear momentum $\boldsymbol{p}$ and the total energy E for a particle with rest mass m_o. Hence, determine the relationship between the linear momentum and the total energy for a particle with zero rest mass. **[6]**

b) A positive pion, with rest mass M_o, decays at rest into a positive muon, with rest mass m_o, and a neutrino, with zero rest mass. The muon emerges along the positive x-axis with linear momentum p_μ. In the rest frame of the pion state:

(*i*) the four-momentum of the initial pion;

(*ii*) the four-momentum of the muon;

(*iii*) the four-momentum of the neutrino. **[6]**

By considering the conservation of linear momentum and energy, and using the relationship between momentum, total energy and rest mass for each particle, determine the energy of the neutrino in the rest frame of the original pion in terms of M_o and m_o. **[8]**

END OF PAPER