B. Sc. Examination by course unit 2010

MTH5106 Dynamics of Physical Systems: MOCK EXAM

Duration: 2 hours

Date and time:

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): R. Tavakol

Question 1 (a) The unit vectors in 2-dimensional Cartesian and Polar coordinates are given by $(\underline{i}, \underline{j})$ and $(\hat{e_r}, \hat{e_\theta})$ respectively.

(i) Find the relation between these unit vectors in the form

$$\hat{e_r} = \underline{i}\cos\theta + \underline{j}\sin\theta, \qquad \hat{e_\theta} = -\underline{i}\sin\theta + \underline{j}\cos\theta$$

(ii) Staring from the definition of position vector $\underline{r} = r\hat{e_r}$ show that the velocity in polar coordinates can be expressed in the form

$$\underline{v} = \dot{r}\hat{e_r} + r\dot{\theta}\hat{e_\theta}$$

(b) Define what is meant by the operators Gradient (∇) and Curl ($\nabla\times$).

Prove the identity

$$\nabla \times \nabla\Phi = 0$$

where Φ is a differentiable scalar function.

[22 marks]

Question 2 A particle of unit mass moves along the x-axis under the influence of the force whose magnitude is given by

$$F(x) = 4x(x^2 - 1).$$

(i) Find an expression for the potential energy $\Phi(x)$.

(ii) Show that $x = 0, \pm 1$ are the three equilibrium points and discuss the stability of each.

(iii) Sketch $\Phi(x)$ as a function of x.

(iv) Using the conservation law of energy in the form

$$\frac{1}{2}m\dot{x}^2 + \Phi(x) = E$$

discuss briefly the possible types of motion that occur, by taking different appropriately chosen values for the total energy E.

(iii) If the particle is placed at the position of stable equilibrium and displaced slightly, find the period of the subsequent oscillations.

[26 marks]

Question 3 The equation of motion of a particle of unit mass is given by

$$\ddot{x} + {\omega_0}^2 x = D \sin \omega t, \tag{1}$$

where ${\omega_0}^2$, D and ω are positive constants.

(i) Discuss briefly the meaning of each term appearing in this equation.

(ii) Calculate the homogeneous and particular solutions for this equation and hence write down the general solution of equation (1).

(iii) Describe what happens to x as $\omega \to \omega_0$. What is this phenomenon called?

(iv) The equation (1) above is then modified to

$$\ddot{x} + 2\gamma\dot{x} + {\omega_0}^2 x = D \sin \omega t.$$

Discuss briefly, in no more than in paragraph, what you expect the effect of the extra term $2\gamma\dot{x}$ to be on the subsequent behaviour of particle.

[26 marks]

Question 4

(i) Using the fact that the acceleration in polar coordinates is given by

$$\underline{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e_r} + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{e_\theta}$$

Show that the orbit of a particle of mass m in the gravitational field of a central body of mass M is given by

$$\frac{d^2u}{d\theta^2} + u = \frac{Gm^2M}{J^2} \equiv \frac{1}{\ell}$$

where $u = 1/r$ and J is the magnitude of the angular momentumi vector.

(ii) State Kepler's 3nd law.

(iii) The semi-major axis of the orbit of Jupiter is 5.2 times that of the Earth. Find its orbital period in years.

[26 marks]

End of Paper