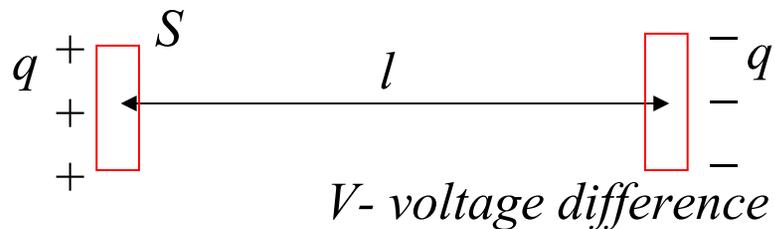


Introduction to Dielectric Properties and Magnetism

At the end of the last lecture we looked at some of the electrical properties of matter and introduces the notions of electric field and electrical conductivity. We shall now look at the interaction of field with matter.



Electric field in vacuum between two plates $E_V = V/l = q/(S\epsilon_0)$, where $\epsilon_0 = 8.85 * 10^{-12} \text{ F/m}$ dielectric permittivity of free space

What is potential? Potential difference is $ql/(S\epsilon_0)$ and the capacity defined as:

$$C = \frac{\text{charge}}{\text{potential}} = \frac{q}{\frac{ql}{S\epsilon_0}} = \frac{\epsilon_0}{l}$$

Dielectric properties

Lets now see what will happen if we fill the space between the plates with a *dielectric* (non-conducting material, we know what will happen if we fill it with conducting material). We will find that *capacity* or *relative permittivity* increased by a factor of ϵ – *dielectric constant* of the material. Since in our simple consideration q is unchanged, then potential difference must change and not due to S as that is the same too. Hence we conclude that overall potential has changed by ϵ and therefore the electric field is changed:

$$E_V = \frac{q}{S\epsilon_0} \rightarrow E = \frac{q}{S\epsilon_0} \frac{1}{\epsilon}$$

This reduction in field is due to *polarisation* of the material. The applied electric field distorts atoms and molecules and orients existing dipoles in such a way as to create a field *opposing* to the applied field (it is as if material is trying to minimise the effect of the external field). Thus the resultant field is reduced.

Dielectric properties cntd

$$E_V = \frac{q}{S} \frac{1}{\epsilon_0}$$

$$E = \frac{q/S - P}{\epsilon_0}$$

We now define polarisation P as the electrostatic moment (ql) per unit volume induced in dielectric. We can rewrite P as follows:

$$P = \frac{q_I l}{V} = \frac{l}{Sl} = \frac{q_I}{S}$$

We can now see that P is equivalent to charge per unit area and hence the effective charge on the plates is $q/S - P$ and

$$E = \frac{1}{\epsilon_0} \left(\frac{q}{S} - P \right) \Rightarrow \frac{q}{S} - P = \epsilon_0 E$$

$$\frac{q}{S} = \epsilon \epsilon_0 E \Rightarrow \epsilon \epsilon_0 E - P = \epsilon_0 E \Rightarrow P = (\epsilon - 1) \epsilon_0 E$$

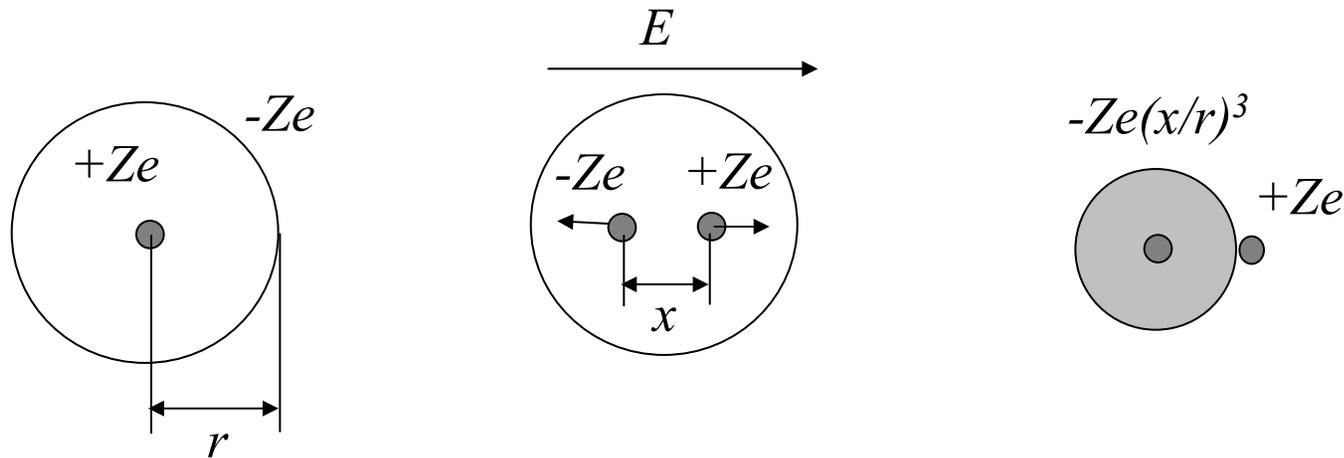
Polarisation of gases

Gases: molecules (atoms) too far apart for interaction. Thus each gas molecule “sees” E and that affects them as follows:

- (i) By distorting the electronic cloud around the nucleus of each molecule; we call the polarisation of this type P_e
- (ii) By stretching or bending bonds of polar molecules; $P_a \sim 0.1P_e$
- (iii) By orienting existing molecular dipoles: P_d

Lets start from (i). As a result of application of electric field E to initially neutral molecule (atom) it will acquire dipole moment $p = \alpha E$, where α is the molecular (atomic) polarizability. Thus for n molecules per m^3 we will obtain $P_e = np = n\alpha E$.

Polarisation of gases: the case of neutral monoatomic molecules



$$F = \frac{(\text{positive charge}) \times (\text{negative charge})}{4\pi\epsilon_0 x^2} = Ze \times Ze \left(\frac{x}{r}\right)^3 \frac{1}{4\pi\epsilon_0 x^2} = \frac{Z^2 e^2 x}{4\pi\epsilon_0 r^3}$$

this balanced by distortion force

$$F = ZeE$$

equating we get

$$E = \frac{Zex}{4\pi\epsilon_0 r^3}, \text{ but } Zex = p \text{ hence } p = 4\pi\epsilon_0 r^3 E$$

$$p = \alpha E \Rightarrow \alpha = 4\pi\epsilon_0 r^3$$

$$P_e = n\alpha E = n4\pi\epsilon_0 r^3 E = (\epsilon - 1)\epsilon_0 E$$

$$\epsilon - 1 = 4n\pi r^3$$

Thus, by measuring some macroscopic electrical properties (such as ϵ) we can get info on microscopic ones (such as r).

Polarisation of gases: the case polar molecules

Apart from polarization polar molecules will show (ii) stretching or bending bonds of polar molecules (e. g. if field is applied at an angle to the initial dipole moment) but $P_a \sim 0.1P_e$, so we shall neglect it and consider

(iii) Orientation of existing molecular dipoles: P_d



No field, no net momentum due to randomization. Switch on field and all dipoles are aligned, then $P_d = np$, and for a gas at STP $n \approx 3 * 10^{25} \text{m}^{-3}$, hence

$P_d \approx 5 * 10^{-4} \text{C/m}^2$, and the field produced is $E_d = P_d / \epsilon_0 = 6 * 10^7 \text{V/m}$

This would be more than enough to ionize most gases, but experiments show that this is not possible. Can you suggest why? The answer is temperature. Thermal motion distorts alignment of dipoles reducing the net dipole moment and hence the electric field. Our rough estimate will not be even achieved at absolute zero due to quantum effect of zero point motion. Can one suggest how the T dependence of net dipole moment would look like?

Polarisation of gases: effect of frequency

Lets also analyse the effect of altering E on electronic polarisation P_e . Assume mass m , charge $-e$ and an electron bound to nucleus by a force with constant k ($F=-kx$). Thanks to Isaac Newton, we have:

$$F = ma = -kx \Rightarrow m\ddot{x} + kx = 0 \text{ or } \ddot{x} + \omega_0^2 x = 0, \text{ where } \omega_0^2 = k / m - \text{natural vib. frequency}$$

Now, if we apply a sinusoidal electric field $E=E_0 \sin \omega t$ we will get:

$\ddot{x} + \omega_0^2 x = F / m = -eE / m = E_0 \sin \omega t$ for which a steady state solution is

$$x = \frac{-eE_0 \sin \omega t}{m} \frac{1}{\omega_0^2 - \omega^2}$$

$$p = -ex = \frac{e^2 E_0 \sin \omega t}{m} \frac{1}{\omega_0^2 - \omega^2} \text{ and we have a resonance effect just like on a swing}$$

$P_e = np$ and we have a resonance effect for ϵ too :

$$\epsilon - 1 = \frac{P_e}{\epsilon_0 E} = \frac{np}{\epsilon_0 E}$$

Dielectric properties of liquid and solids

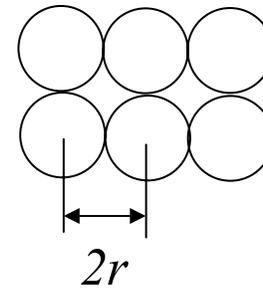
(i) Weak interaction (non-polar molecules)

$$p = 4\pi\epsilon_0 r^3 E$$

where r is atomic radius. For cubic structure in 3D

$$n = \left(\frac{1}{2r}\right)^3$$

$$P = np = \frac{\pi\epsilon_0}{2} E, \text{ but } P = (\epsilon - 1)\epsilon_0 E \Rightarrow (\epsilon - 1) = \frac{P}{\epsilon_0 E} = \frac{\pi}{2}$$



This is the case for many solids/liquids: $\epsilon \approx 2-3$. We should point out that in a crystal which has a well defined structural anisotropy ϵ will depend on crystal direction.

Dielectric properties of liquid and solids, cntd.

(ii) Strong interaction. If we consider the case of interacting dipoles the picture becomes a little more complicated. Not only there will be a net polarisation P due to applied electric field E , but each dipole with $p=4\pi\epsilon_0 r^3 E$ will produce field $E=p/4\pi\epsilon_0 r^3$ at a distance r . This field is comparable to the applied field itself and increases polarisation effect.

$$P = \alpha E$$

$$\alpha = \frac{\pi\epsilon_0}{2} \text{ for weak interaction}$$

$$\text{now we will have } \frac{\epsilon - 1}{\epsilon + 2} = \frac{n\alpha}{3\epsilon_0}$$

So, α now depends on ϵ and so does polarisation which can now be anything from the case of weak interaction (if $\epsilon \approx 2-3$) to large dipole interaction if $\epsilon = 70$.

Few words on frequency dependence of P in solids. It has somewhat similar effect in liquids to that of gases, but more sluggish due to stronger interaction. Consequently, has virtually no effect in solids as it is difficult or impossible to rotate a bound polar molecule.

Summary

Polarisation due to electric field occurs in the following ways:

- a. Distortion of electron cloud; all atoms (molecules) show this effect
- b. With polar molecules stretching or bending of bonds can produce a small contribution to the polarisation
- c. With polar molecules orientation of the molecule may produce very large polarisation, but the field has to compete with randomisation effect of T.

T has no effect on (a) since this involves electronic properties and electron energies much higher than thermal energy ($1\text{eV} \approx 11000\text{K}$). T resists increase of polarisation in (b) and (c), still polarisation can not increase indefinitely as we lower T as materials turn into solid and it may become impossible to rotate molecular dipoles.

Frequency has marked effect on all types of polarisation. Orientation of whole dipoles becomes impossible at about 10^{12}Hz . Stretching/bending becomes unable to follow field at 10^{14}Hz . Only electronic polarisation remains up to 10^{15-16}Hz . This is optical range and therefore the only part of dielectric mechanism that plays part in refractive index of the material.

In the solid state the picture is further complicated by strong interatomic interactions which can greatly increase polarisation effect in the material.