Answer all SIX questions in SECTION A and THREE from SECTION B.
The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may need:

Permeability of free space $4\pi \times 10^{-7}$ H m^{-1}.
Permittivity of free space 8.854×10^{-12} F m^{-1}.
Speed of light *in vacuo* 3×10^{8} m s^{-1}.
Charge on electron 1.6022×10^{-19} C
Mass of electron 9.1094×10^{-31} kg
For any vector field **A**: $\nabla . \nabla \times \mathbf{A} = 0$
and $\nabla \times \nabla \times \mathbf{A} = \nabla \left(\nabla . \mathbf{A} \right) - \nabla^2 \mathbf{A}$.

SECTION A

1. Write down the relationship between the electric field strength **E**, the electric dispacement **D** and the polarisation **P**. State the units for each quantity. Write down the equivalent relationship between the magnetic vector fields **B**, **H** and **M**. Give the name of each of these fields. [2]

   Explain very briefly why only two of these six fields are needed to fully describe electromagnetic effects in vacuum. Which ones are they? [1]

   Give brief descriptions of the physical properties of the materials which give rise to the quantities **P** and **M**. [3]

2. Write down the expression for the e.m.f. induced in a closed conducting circuit C by the rate of change of the magnetic flux Φ_c which passes through it. Explain how this is related to the integral form of the Faraday law. [3]

   Use the appropriate theorem to derive the differential form of the Faraday law from the integral form. [3]

PLEASE TURN OVER

3. Show, starting with the appropriate Maxwell equation, that normal components of **D** are continuous across a plane boundary between different media at which there are no surface charges. [5]

Explain briefly why this boundary condition leads to the conservation of lines of **D.** **[2]**

4. A toroidal magnet has a primary coil with N turns carrying I amps wound on a ferromagnetic yoke with minor radius r and major radius $\simeq R$. Use the appropriate law to derive an approximate expression for the value of H inside the material. Briefly explain the assumptions made. [3]

With the aid of a sketch, state briefly what needs to be added to such a toroid in order to measure changes in the value of B inside the material. Hence explain how the major hysteresis loop for the material of the yoke can be established. [4]

5. Draw a ray diagram for light falling onto a plane glass surface from vacuum. Label the angles of incidence, reflection and refraction. [2]

Assuming that incoming and outgoing waves have the form

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \exp i(\mathbf{k}.\mathbf{r} - \omega t)$$

derive Snell's law for refraction and show that the angles of incidence and reflection must be equal. State any extra assumptions you need to make. [5]

6. Given the equation $\nabla^2\mathbf{E} - \mu\sigma\frac{\partial \mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$, show that plane electromagnetic waves of the form given in question 5 above, in a uniform conducting medium, obey the dispersion relation $k^2 = \mu\varepsilon\omega^2\left(1 + i\sigma/\varepsilon\omega\right)$. Define all the symbols in these two equations. State briefly what this implies for the quantity k in a good conductor. [4]

Show in the limit when $\sigma \to 0$ that the phase and group velocities derived from this relation are equal. How are they related to the velocity of light in vacuum? [3]

CONTINUED

SECTION B

7. We can treat each turn of a long solenoid as a distinct circuit i carrying current I_i, with magnetic flux Φ_i passing through it. Given that the total magnetic energy stored in a system of circuits is $W = \frac{1}{2}\sum_i I_i \Phi_i$, show that the energy per unit volume stored in a long solenoid wound over a uniformly magnetized cylindrical core is $U_m = \frac{1}{2} HB$. [8]

A material whose magnetic properties are dominated by the Curie law has $\chi_m = \frac{N m_0^2 \mu_0}{kT} + \chi_0$. What does each of the symbols in this expression represent? Write down the expression relating **B**, **H** and χ_m in a linear isotropic material. What kind of behaviour is governed by the Curie law; paramagnetic, diamagnetic or ferromagnetic? Briefly describe how the magnetic properties of each of these types of magnetic material varies with temperature. [4]

A piece of material whose properties are dominated by the Curie law has magnetic susceptibility 0.9×10^{-4} at 300 K and 2.9×10^{-4} at 100 K. What will its susceptibility be at 0.003 K? [4]

If the material is kept in a region with constant uniform magnetic field strength H, by what factor will the stored magnetic energy in the sample increase when the temperature is reduced from 300 K to 0.003 K? [4]

[Edited from the original version of the question]

8. Explain the relation between the continuity equation $\nabla . \mathbf{J}_f = -\frac{\partial \rho_f}{\partial t}$ and the conservation of charge. [3]

The Ampere law in its original form was $\nabla \times \mathbf{H} = \mathbf{J}_f$. Show that this equation is inconsistent with the continuity equation. Use the Gauss law to show that an extra term, the displacement current density $\frac{\partial \mathbf{D}}{\partial t}$, must be added to the right hand side of the Ampere law to make the two equations compatible. [6]

By taking the curl of the modified Ampère law, and justifying all assumptions made, derive the wave equation $\nabla^2 \mathbf{H} = \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2}$. [5]

Demonstrate from the steps of your derivation that the displacement current term must be present to obtain a second order differential wave equation. [2]

In a medium where the relative permittivity is 2.5 and the relative permeability is 0.95, what is the speed of light? [4]

9. The dispersion relation for waves in a plasma is $k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$, where $\omega_p = \sqrt{\frac{N_e e^2}{\varepsilon_0 m}}$. Define all of the quantities in these two expressions. Give a brief qualitative explanation of ω_p. [3]

If $\omega > \omega_p$ demonstrate that the group velocity $\mathrm{v_g}$ and the phase velocity $\mathrm{v_p}$ for such waves satisfy the equation $\mathrm{v_g v_p} = c^2$. State briefly how this can be reconciled with expectations from the special theory of relativity. [5]

Show that if $\omega < \omega_p$ electromagnetic waves will be absorbed as they pass through the plasma, with attenuation length $L = \frac{c}{\omega}\sqrt{\frac{1}{\left(\omega_p^2/\omega^2 - 1\right)}}$. [6]

A (fictional) Vogon spaceship sets up a uniform plasma barrier 1m thick to defend itself against incoming electromagnetic radiation at 1 MHz. What is the minimum electron density required for such a plasma to have any shielding effect? [3]

How much must the electron density be increased beyond the minimum in order to attenuate the amplitude of the incoming radiation by a factor of e^{-2}? [3]

10. Briefly explain the physical meaning of the Poynting vector $\mathbf{N} = \mathbf{E} \times \mathbf{H}$ for time-varying fields. What are its units? [3]

Show that for plane electromagnetic waves the average of the Poynting vector over a whole number of cycles is given by $<\mathbf{N}> = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\hat{\mathbf{k}}$. Define all of the quantities in this expression. [7]

Show, by summing over photons or by other arguments, that the pressure exerted by an electromagnetic wave which is reflected normally from a surface is $2 < N > / c$. [5]

What is the minimum area required for a perfectly reflecting solar sail which would drive a 1000 kg spaceship outward from the Earth's orbit with an acceleration of 10 m s^{-2}? [5]

[Energy flow in sunlight at the Earth's orbit = 1300 W m^{-2}.]

CONTINUED…………

11. Why are tangential components of the electric field vector **E** at the conducting surfaces of a waveguide required to be zero? Demonstrate from this requirement that plane TEM waves at radio frequencies cannot be transmitted in a rectangular waveguide. [4]

Explain with diagrams (but without full derivation) how TE_{lm} and TM_{lm} waves can satisfy the above requirement. [3]

The waveguide equation for such waves is $\frac{\ell^2}{a^2} + \frac{m^2}{b^2} = \frac{k_0^2 - k_g^2}{\pi^2}$. Define all of the quantities in this expression and use it to derive a formula for the cutoff frequency of the guide. [6]

We wish to make a waveguide in which the three lowest allowed frequencies all have $m = 0$. What constraint does this place upon the ratio a/b? [7]

END OF PAPER