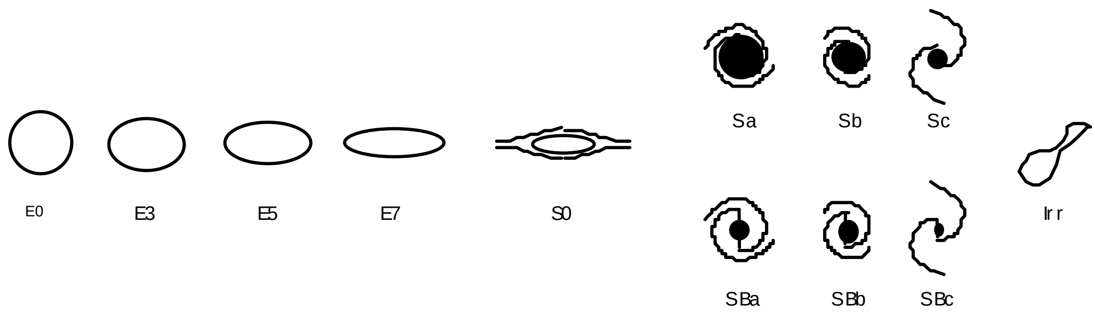
Queen Mary & Westfield College

680/0412 Physics of Galaxies (BSc) Summer 2004: Answers to Questions

SECTION A (Answer 5 questions. Each question carries 8 marks.)

A.1 Hubble's scheme is

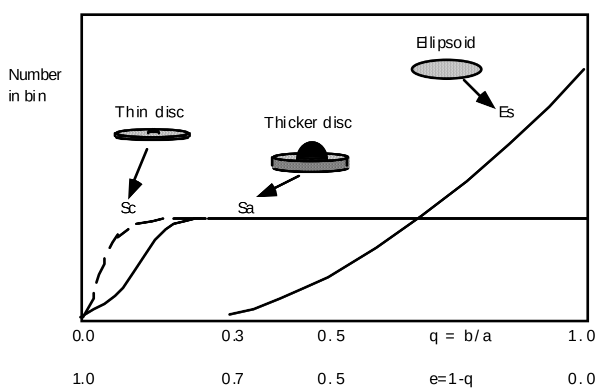


[5 marks]

Figure shows that elliptical galaxies are redder than spirals and that spirals get bluer as one goes along the Hubble sequence. As hot young stars are blue whilst, on the whole, red stars are old this suggest that ellipticals contain predominantly old stars whilst spirals contain increasingly more young stars (more recent star formation) as we go from S*a/SBa* to S*c/SBc.* **[3 marks]**

A.2 The flattest elliptical galaxies seen are E7, corresponding to the value 0.7 for *e*. The ellipticity of a flat disc, viewed at an angle θ to its normal, is given by $e = 1 - \cos\theta$. So $e_{maximum}$=0.7 corresponds to $(\cos\theta)_{minimum} = 0.3$ or $\theta_{maximum} = \cos^{-1}(0.3) = 72.5°$. Hence, if all elliptical galaxies were flat discs inclined at various angles to our line of sight, they would have to conspire to be so inclined by no more than about 72.5°. [Exact value of 72.5 not required] **[5 marks]**

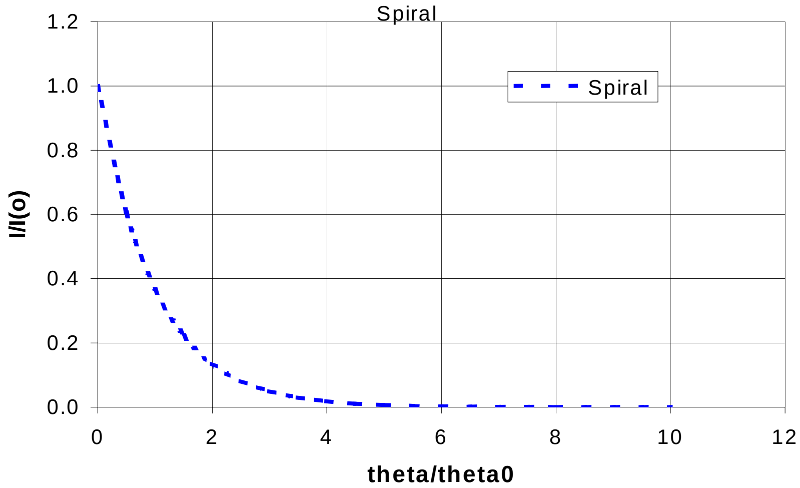
[**OR** student can sketch the distribution of elliptical galaxies as a function of the ratio of the smaller to the longer dimension].



If all such galaxies were (infinitesimally) flat discs, then the distribution would be a horizontal line, which is not in keeping with this prediction. **[*alternative 5 marks*]]**

Spirals show a much more uniform range of axial ratio/ellipticity, the highest values of e being only limited when the thickness of the disc begins to play a part. This shows that spirals are (thin) discs inclined at various angles to the line of sight. **[3 marks]**

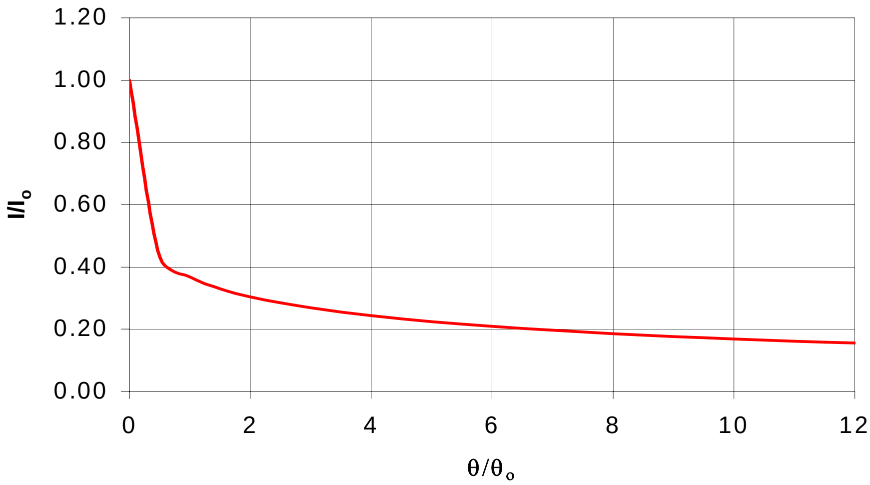
A.3 The surface brightness of a galaxy is its flux density per unit solid angle, as a function of position in the galaxian image. **[2 marks]**



[2 marks]

The surface brightness of the nuclear bulge of an elliptical galaxy, as a function of angular distance θ from its centre, is given by the 'de Vaucoluers' profile

$$I_{\mathrm{E}}(\theta) = I_{\mathrm{E}}(0)\exp\left[-\left(\frac{\theta}{\theta_{\mathrm{oE}}}\right)^{1/4}\right],$$ $I_{\mathrm{E}}(0)$ and θ_{oE} being constants. **[2 marks]**



[2 marks]

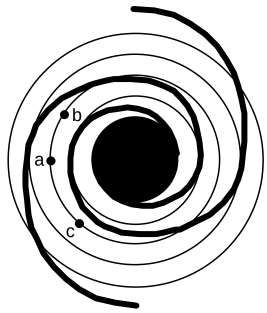
A.4 The masses of galaxies deduced from dynamical (gravitational) arguments are much larger than those that would be deduced from using the luminosity and the M/L ratio of stars. Cool Interstellar matter, black holes, planets, brown dwarfs, etc cannot resolve the discrepancy.
[3 marks]

For spiral galaxies the dynamical argument is from the rotation curves which are flat (v(r) = constant at large r) indicating that mass exists out to large r, yet the surface brightness falls off at large r. Hence there is matter which is not producing light in spiral galaxies. **[2 marks]**

For elliptical galaxies the Virial theorem is used with observed mean square velocity dispersion to come to a similar conclusion. **[2 marks]**

Applying the Virial theorem to clusters of galaxies similarly indicates more mass in the clusters than contained in the individual galaxies. **[1 marks]**

A.5



Consider a star or cloud of gas at position *a*, roughly equidistant from both arms; it will therefore suffer no net radial force. **[1 marks]**

A star or cloud of gas at *b* will be attracted outward by the extra material in the nearer – outside – arm and will move to a slightly larger radius. **[1 marks]**

Conserving its angular momentum, it will slow down a little in its orbit and will tend to linger in the vicinity of the arm. The material bunched in this way gives rise to the additional potential. **[3 marks]**

When a star eventually leaves a spiral arm, as at *c*, it is attracted inwards and speeds up, so regaining its initial speed. In this way, the spiral perturbation is maintained. **[3 marks]**

A.6 $\tau_{ff} < \tau_s$ is the condition for collapse. **[2 marks]**

If the cloud is to prevent itself from collapsing, it must set up pressure gradients within itself before collapse starts. It can only do this by sending pressure wave "messages" at the speed of sound u_s which must be able to cross the cloud (in time $\tau_s \sim \ell/u_s$) before the cloud has had time to collapse (in a free fall time τ_{ff}, the time it would take a cloud to collapse if no internal forces, caused by pressure gradients, were to come into effect). Conversely if $\tau_{ff} < \tau_s$ collapse is inevitable. **[3 marks]**

Consider a cloud of given size l which is initially stable [$\tau_{ff})_{init} > \tau_s)_{init}$]. If τ_{ff} decreases, or τ_s increases, sufficiently below/above their initial values the condition will no longer be met and the cloud will become unstable to collapse. τ_{ff} depends on density$^{-1/2}$ and τ_s less strongly on density (not required: through u_s depends on density$^{(\gamma-1)/2}$, where $\gamma>1$), so if the cloud density increases τ_{ff} will decrease and the cloud may now collapse. When a cloud of critical size enters a shock front, it will be compressed and, if the increase in density is sufficient, it will collapse.

[3 marks]

A.7 a) Satellites of our Galaxy include *two irregular* galaxies (Large & Small Magellanic Clouds), together with *several other dwarfs*, within *100 kpc*. **[3 marks]**

b) The Local Group consists of *three spirals* (the Galaxy, M31 and M33) and some *25 dwarfs* out to about *600 kpc*. **[3 marks]**

c) The Virgo Cluster, contains *thousands* of *elliptical and spiral* galaxies. The centre of the cluster is *13-20 Mpc* away. **[2 marks]**

N.B. Credit will be given for the (rough) numbers of galaxies, type and location, rather than for accuracy of their names.

A.8 The Eddington limit is obtained by balancing the inward gravitational force on the accreting material with the outward force of the photons on the same material. **[2 marks]**

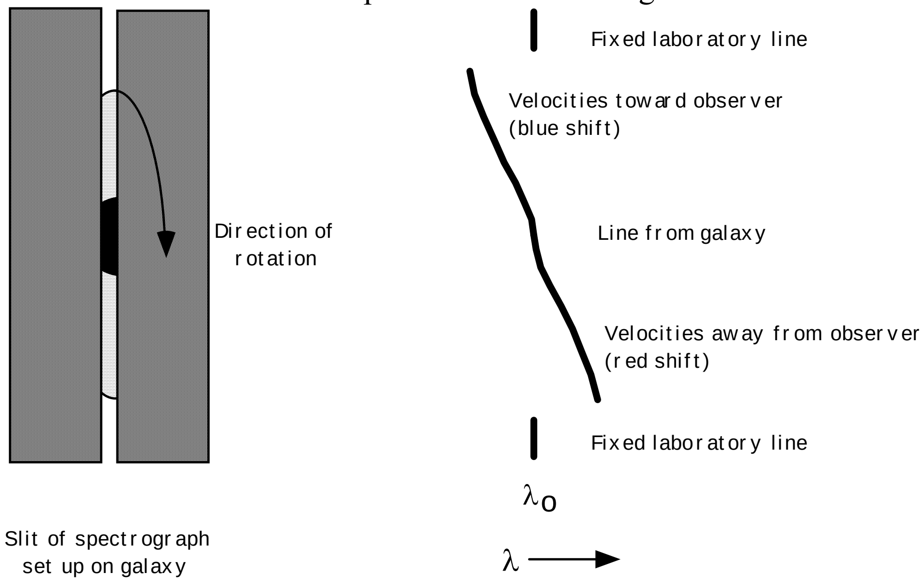
The accreting material is presumably fully ionised, consisting therefore of electrons and protons. **[1 marks]**

The outward force is controlled by the cross-section for the scattering of photons by electrons - Thomson scattering –which is very much greater than that for scattering by protons. The outward force on the material is therefore dominated by Thomson scattering. **[2 marks]**

The gravitation force is predominately on the protons and is some 2000 times - the ratio of their masses- more than that on the electrons. The inward force is therefore dominated by the gravitational force on the protons. **[2 marks]**

The electrons and protons are "glued" together electrostatically so the outward force on the electrons is communicated to the protons. **[1 marks]**

B.1 The answer should be a précis of the following:



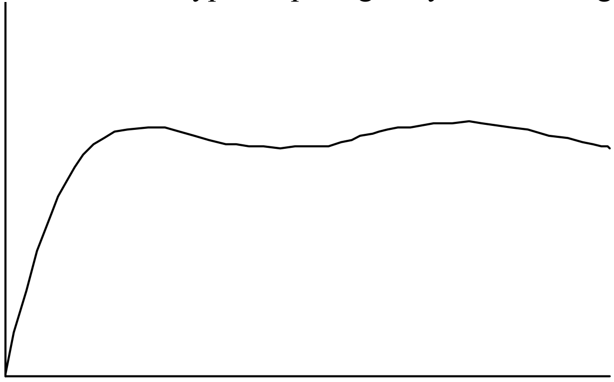
Bits of answer in [] are not required for full marks.

The figure shows schematically how long-slit spectra are used to measure a galaxy's rotation curve in the optical. The slit of the spectrograph is placed along the major axis of the galaxy as shown. The resultant spectrum of a single line [of rest-wavelength λ_o] is shown on the right, [relative to the same line produced by a laboratory source]. The parts of the galaxy which are approaching the observer give rise to blue shifted lines whilst those which are receding give rise to red-shifted lines. The overall effect is to produce the curved spectral line shown in the figure. [For simplicity, I have assumed that the galaxy as a while is at rest with respect to the Earth. In practice, the centre of the line would also be red-shifted with respect to the laboratory line because of the galaxy's recession.] The Doppler shift can be converted directly into velocity along the line of sight. **[4marks]**

In order to see clearly from what part of a galaxy a particular line originates, we should prefer the galaxy to be as face-on to us as possible. Unfortunately, the material in a face-on galaxy ($i=90^o$) has no line-of-sight component of velocity ($v \cos i=0$) and so there is no Doppler shift! The *maximum* shift is obtained principle from an edge-on galaxy but there is a problem with such an orientation. The line-of-sight passes through material at a range of radii and is hence moving at a range of velocities, making interpretation difficult. We compromise by choosing galaxies which are sufficiently inclined to allow us to resolve which radius in the disc the lines are coming from, whilst still giving appreciable components of velocity in the line of sight. **[2 marks]**

[Also we have to look *through* the material of an edge-on disc; many disc galaxies have dust lanes which obscure starlight.]

The rotation curve for a typical spiral galaxy is flat at large r.



[5 marks]

Rigid body rotation would imply that all the material is somehow held together in a fixed relative orientation, and no mechanism for doing this is known. **[2 marks]**

For a spherically symmetric distribution of matter, the circular velocity $\Theta(r)$ of a star of mass m at distance r from the centre of the galaxy is given by

$$\frac{m\Theta^2(r)}{r} = \frac{GM(r)m}{r^2}$$

so that, as $\Theta(r) = \Theta_o = \text{constant}$ is observed at large r,

then $$M(r) = \frac{\Theta_o{}^2}{G} \times r.$$ **[6 marks]**

In general, the mass $dM(r)$ contained between r and $r+dr$ is given by

$$dM(r) = 4\pi r^2 dr \times \rho(r)$$

so that $$M(r) = \int_0^r dM(r') = 4\pi \int_0^r \rho(r') r'^2 dr'.$$ **[2 marks]**

If $$\rho(r) = \rho_o \left(\frac{r}{r_o}\right)^{\alpha},$$

then $M(r)$ is given by

$$M(r) = \frac{4\pi r_o \rho_o}{r_0{}^{\alpha}} \int_o^r r'^{2+\alpha} dr'$$

$$= \frac{4\pi}{(3+\alpha)} r_o{}^{-\alpha} \rho_o r^{3+\alpha}$$

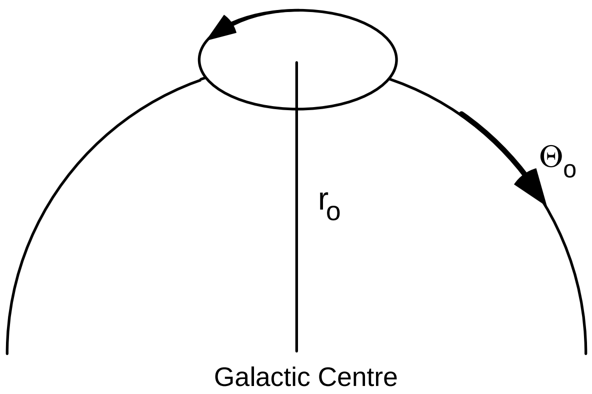
[4 marks]

Comparing this result with the previous expression for $M(r)$, we see that, for consistency, we must have $3+\alpha = 1$ or $\alpha = -2$. **[2 marks]**

If the density were continue to fall only as r^{-2}, then $M(r)$ would remain proportional to r indefinitely and the mass of the galaxy would be infinite! The density must, therefore, eventually fall off more rapidly than r^{-2}. **[3 marks]**

B.2

(a)



Consider a star in circular motion about the centre of a galaxy and suppose it is pushed radially outward a little *while keeping its angular momentum constant.* As the star moves out, its circular velocity will decrease from the unperturbed value of Θ_O. It will therefore be moving around the centre of the galaxy *slower* than other stars which are in circular orbit at this slightly larger radius and which therefore have velocity Θ_O in the θ-direction. Relative to these stars, therefore, the perturbed star will appear to be moving backwards. Furthermore, with its reduced velocity, it will not have sufficient centrifugal force to overcome the gravitational force and it will tend to drop in towards the centre of the galaxy. As it drops down below its original circular orbit at r_O, conservation of angular momentum means that it will now be going *too fast* with respect to stars in circular orbit. It will be overtaking them and, now having too much centrifugal force, will move back out. This motion is called epicyclic motion. **[6 marks]**

(b) If flat $\Theta(r) = \Theta_o = \text{constant}$, and $\dfrac{d\Theta}{dr} = 0$

and $A(r) = +\dfrac{1}{2}\left[\dfrac{\Theta_o}{r} - 0\right] = +\dfrac{1}{2}\left[\dfrac{\Theta_o}{r} + 0\right] = -B(r) = \dfrac{1}{2}\dfrac{\Theta_o}{r}$

Using these expressions for $k(r)$, $A(r)$ and $B(r)$, we have

$$\kappa(r) = \sqrt{-4\left[-\frac{1}{2}\frac{\Theta_o}{r}\right]\left[\frac{1}{2}\frac{\Theta_o}{r} + \frac{1}{2}\frac{\Theta_o}{r}\right]} = \sqrt{2\left(\frac{\Theta_o}{r}\right)^2},$$

But $\dfrac{\Theta_o}{r} = \Omega(r)$,

the angular frequency of the material in circular orbit at radius r, so that

$$\kappa(r) = \sqrt{2}\,\Omega(r)$$ **[4 marks]**

The condition for closure of orbits in an inertial frame is that during the time, T, in which a whole number, q, of epicycles are completed a whole number, p, of galactic orbits are also completed. So $T = p\dfrac{2\pi}{\Omega(r)} = q\dfrac{2\pi}{\kappa(r)}$ and hence $\dfrac{\Omega(r)}{\kappa(r)} = \dfrac{p}{q}$, where p & q are integers, is required for closed orbits. In a galaxy with a completely flat rotation curve $\frac{\Omega(r)}{\kappa(r)} = \frac{1}{\sqrt{2}}$, and as $1/\sqrt{2}$ cannot be expressed as the ratio of two integers, the orbits are not closed in this case. **[6 marks]**

(c) If the turbulent velocity is set equal to zero, the dispersion relation becomes

$$m^2(\Omega_p - \Omega(r))^2 = \kappa(r)^2 - 2\pi G\sigma_o \,|\, k \,|$$

which can be re-written as

$$2\pi\sigma_0 G|k| = \kappa(r)^2 - m^2(\Omega_p - \Omega(r))^2 .$$

The left-hand side of this equation is obviously non-negative so that we must have

$$(\Omega_p - \Omega(r))^2 \le \frac{\kappa(r)^2}{m^2}$$

so

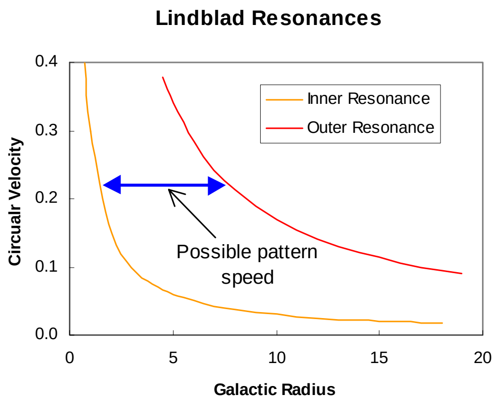
$$-\frac{\kappa(r)}{m} \le (\Omega_p - \Omega(r)) \le +\frac{\kappa(r)}{m}$$

or

$$\Omega(r) - \frac{\kappa(r)}{m} \le \Omega_p \le \Omega(r) + \frac{\kappa(r)}{m} .$$

[7 marks]

(d) In a frame rotating at Ω_p orbits are closed (in this frame) when $\frac{(\Omega_p - \Omega(r))}{\kappa(r)} = \pm\frac{p}{q}$. The section describes this case with $p=1$ and $q=m$ so resonances do occur.



[Figure is not required]

We know that, if we repeatedly "hit" something at its resonant frequency, the oscillations tend to build up and the object may destroy itself. Spiral density waves can therefore only exist between the ['Lindblad'] resonances given by the above relation. **[7 marks]**

B.3 The flux density S received from a galaxy with luminosity L at distance r is given by $S = \frac{L}{4\pi r^2}$ so that the distance $r(S)$ of a galaxy with luminosity L measured with flux-density S is given by $r(S) = \left(\frac{L}{4\pi S}\right)^{1/2}$, which increases as S decreases. Hence, the maximum distance $r_{\max}$ of a galaxy with luminosity L a survey with minimum flux density $S_{\lim}$ is given by $r_{\max} = \left(\frac{L}{4\pi S_{\lim}}\right)^{1/2}$. **[4 marks]**

If it is to appear in the survey of solid angle Ω (whole sky is 4π), a galaxy of luminosity L must be within a volume $V(L)$ of space given by $V(L) = \frac{4\pi r_{\max}^3}{3}\frac{\Omega}{4\pi}$. **[3 marks]**

There are $\phi(L)dL$ galaxies per unit volume with luminosities in the range L to $L + dL$ so that the number $N(L)dL$ of such galaxies appearing in the survey is given by

$$N(L)dL = V(L) \times \phi(L)dL = \frac{\Omega}{3}\left(\frac{L}{4\pi S_{\lim}}\right)^{3/2}\phi(L)dL = \frac{\Omega}{3}(4\pi S_{\lim})^{-3/2} L^{3/2}\varphi(L)dL.$$ **[3 marks]**

The total number N of galaxies per unit volume with luminosities between L_1 and L_2 in the sky is given by $N = \int_{L_1}^{L_2}\phi(L)dL = KL_*^{\alpha}\int_{L_1}^{L_2} L^{-\alpha}dL = KL_*^{\alpha}\frac{1}{1-\alpha}\left[L^{1-\alpha}\right]_{L_1}^{L_2}$. **[3 marks should be 8]**

Or if they assume it is galaxies in the survey (harder) credit is still given

The total luminosity L generated per unit volume is given by

$$L = \int_{L_1}^{L_2} L\phi(L)dL = KL_*^{\alpha}\int_{L_1}^{L_2} L^{1-\alpha}dL = KL_*^{\alpha}\frac{1}{2-\alpha}\left[L^{2-\alpha}\right]_{L_1}^{L_2}$$

[5 marks – but not in question!]

Substituting numerical values into the equation for N, we get

$$N = 4\times10^{-13}\times\left(6\times10^{9}\right)^{1.4}\times\frac{1}{-0.4}\left[\left(10^{11}\right)^{-0.4} - \left(10^{8}\right)^{-0.4}\right]$$

$$= 4\times10^{-13}\times\frac{12.29\times10^{12.6}}{0.4}\left[10^{-3.2} - 10^{-4.4}\right] = 0.03\,\text{Galaxies Mpc}^{-3}$$

[6 marks - too much for a bit of maths]

For the power-law,

$$N \propto \left[L^{1-\alpha}\right]_{L_2}^{L_1} = \left[L_1^{-0.4} - L_2^{-0.4}\right]$$

and $L \propto \left[L^{2-\alpha}\right]_{L_1}^{L_2} = \left[L_2^{0.6} - L_1^{0.6}\right]$.

N therefore diverges as the lower luminosity limit L_1 tends to zero whilst L tends to infinity as the upper limit L_2 tends to infinity. The power-law cannot, therefore, continue indefinitely to either low or high luminosities. **[4 marks]**

The Schecter form is $\phi(L) = K'\left(\frac{L}{L_*}\right)^{-1.1} e^{-(L/L_*)}$ [α will do instead of 1.1] which because of its exponentially decreasing term makes the integrals finite. **[2 marks]**

B.4

(a) The energy must derive from conversion of mass to energy in some way. As the energies are so vast, and produced in such small regions, we should seek the most efficient means of converting stellar mass to AGN energy. Nuclear burning is only about 0.7% efficient. On the other hand conversion of mass to energy has an efficiency of ~8% (=1/12 see below). Thus the rate of consumption of stars (and so rate of replenishment if in equilibrium) is less of a problem for grav'nal accretion. **[5 marks]**

(b) From the virial theorem we have $2\Delta T + \Delta\Omega = 0$ so $\Delta T = -\frac{1}{2}\Delta\Omega$. **[1 marks]**

Conservation of energy tells us that $T + \Omega + E = \text{constant}$, where E is the energy radiated,

so that $\Delta T + \Delta\Omega + \Delta E = 0$ **[3 marks]**

Substituting for ΔT in the conservation of energy equation, $-\frac{1}{2}\Delta\Omega + \Delta\Omega + \Delta E = 0$

Which simplifies to $\Delta E = -\frac{1}{2}\Delta\Omega$.

The energy radiated is therefore equal to half the loss in potential energy. **[2 marks]**

(c) The potential energy is -GMm/R. **[1 marks]**

From the above result, we have for the energy ΔE radiated in time Δt when a mass Δm falls to within R of the central source of mass M,

$$\frac{\Delta E}{\Delta t} = \frac{1}{2}\frac{GM}{R}\frac{\Delta m}{\Delta t}.$$ **[3 marks]**

Taking the limit as Δt goes to zero, we have

$$L \equiv \frac{dE}{dt} = \frac{1}{2}\frac{GM}{R}\frac{dm}{dt} = \frac{1}{2}\frac{GM\dot{m}}{R}.$$ **[2 marks]**

We are given that the last stable orbit is at three Schwarzschild radii $R_{Schwarzschild}$ from the "centre" of the hole where $R_{Schwarzschild} = \frac{2GM}{c^2}$ so substituting for R in the above result, we get $L_{\max} = \frac{1}{2}\frac{GM\dot{m}}{\left(3\times 2GM/c^2\right)} = \frac{1}{12}\dot{m}c^2$ **[2 marks]**

(d) If the source varies in a time τ, the size R of the region responsible for this emission must satisfy the inequality $R \le c\tau$. **[2 marks]**

Since the minimum size of the region must exceed the diameter of the last stable orbit, we must have

$$12\frac{GM}{c^2} \le R \le c\tau,$$ **[2 marks]**

so that $M \le \frac{1}{12}\frac{c^3\tau}{G}$, **[2 marks]**

(e) The luminosities of AGN range from about 10^{33} W to 10^{40} W, so that the lower limit on masses range from about $\sim 10^2$ to $\sim 10^9$ Mo. **[2 marks]**
Variability is on the timescale of $\sim 10^{-2}$ hr to $\sim 10^2$ hr, giving upper limits of between $\sim 10^6$ to $\sim 10^{10}$ Mo. **[2 marks]**
This would suggest (ignoring any correlation between L and variability) that at least some masses must be in the range 10^9 to $\sim 10^{10}$ Mo. **[1 marks]**
