BSc/MSci EXAMINATION

PHY412 Physics of Galaxies

Time Allowed: 2 hours 15 minutes

Date: 27 May 2004

Time: 14.30 – 16.45

Candidates should answer FIVE of the eight questions in Section A, each of which carries 8 marks, and TWO of the four questions in Section B, each of which carries 30 marks; SEVEN questions should be answered altogether. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.

Miscellaneous Data

Velocity of light	c	3.00×10^{8}	m s^{-1}
Gravitational constant	G	6.67×10^{-11}	N m^{2} kg^{-2}
Boltzmann constant	k	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	σ	5.67×10^{-8}	W m^{-2} K^{-4}
Thomson cross-section	σ_T	6.65×10^{-29}	m^{2}
Mass of proton	m_p	1.67×10^{-27}	Kg
Mass of sun	M_{sun}	1.99×10^{30}	Kg
Luminosity of sun	L_{sun}	3.83×10^{26}	W
Hubble constant	H_o	$h \times 100$	km s^{-1} Mpc^{-1}
Astronomical unit	AU	1.50×10^{11}	m
Parsec	pc	3.09×10^{16}	m
Year	y	3.16×10^{7}	s

Do not turn to the first page of the question paper until instructed to do so by the invigilator.

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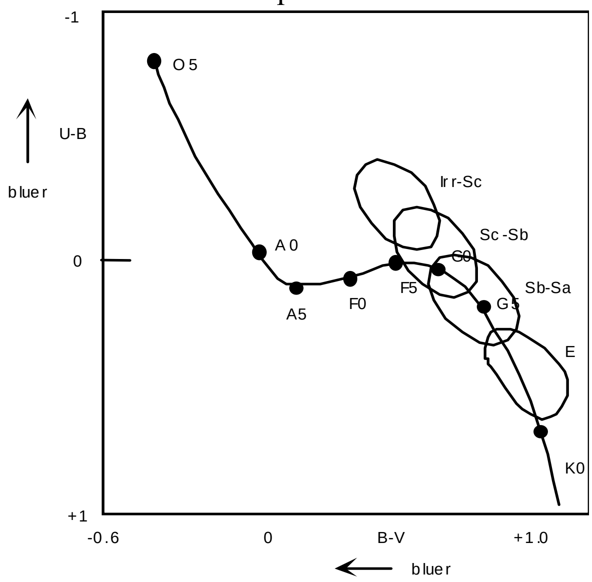
SECTION A (Attempt <u>5 questions</u> from the 8 in this Section. Each question carries 8 marks.)

A.1 Sketch Hubble's scheme for the classification of galaxies. **[5 marks]**

The figure below shows schematically a colour-colour plot of various types of galaxies together with that of the stellar main sequence.



Interpret this plot in terms of the stellar populations and recent rate of star formation, of galaxies. **[3 marks]**

A.2 Explain the statistical evidence for rejecting the hypothesis that elliptical galaxies are flat discs inclined at various angles to the line of sight. **[5 marks]**

Discuss **briefly** the equivalent data for spiral galaxies. **[3 marks]**

A.3 Define what is meant by the *surface brightness I* of a galaxy. **[2 marks]**

The surface brightness $I_d(\theta)$ of the discs of most spiral galaxies, as a function of θ, is well described by the function

$$I_d(\theta) = I_d(0)\exp\left(-\frac{\theta}{\theta_0}\right),$$

where θ is the angular distance from their centre, and θ_0 is a constant.

Sketch $I_d(\theta)/I_d(0)$. **[2 marks]**

Write down the function which describes the surface brightness $I_e(\theta)$ of most *elliptical* galaxies, as a function of angular distance θ from their centres. **[2 marks]**

Sketch $I_e(\theta)/I_e(0)$. **[2 marks]**

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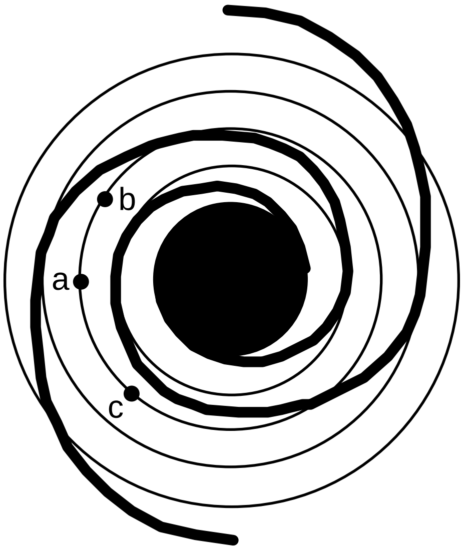
A.4 Explain briefly the evidence for the existence of *dark matter* including the examples of spiral galaxies, elliptical galaxies, and clusters of galaxies. **[8 marks]**

A.5 The figure shows schematically a two-armed spiral galaxy in which *a*, *b*, and *c* represents three bits of disc material in quasi-circular orbit about the centre of the galaxy. Use this figure to give a qualitative description of how spiral density waves maintain themselves.



[8 marks]

A.6 State and briefly explain the Jeans condition for collapse of an interstellar cloud of gas in terms of its free-fall time τ_{ff} and the time τ_s taken for sound to cross the cloud. **[5 marks]**

Explain briefly and qualitatively why the entry of a cloud of gas into the spiral arm of a galaxy can lead to star formation. **[3 marks]**

A.7 Describe briefly the types, numbers and typical approximate distances of galaxies which are

(a) satellites of our Galaxy,

(b) members of the Local Group

(c) members of the Virgo Cluster. **[8 marks]**

A.8 The Eddington luminosity $L_{Eddington}$ of a source of mass M is given by

$$L_{Eddington} = 4\pi \frac{GMm_p c}{\sigma_T} .$$

Explain **briefly and without derivation** the physical basis of the Eddington Luminosity including explaining why the *proton* mass appears in this equation although it is Thomson scattering by *electrons* which limits the luminosity. **[8 marks]**

Please see the next page.

Section B (Attempt 2 questions from the 4 in this Section.
Each question carries 30 marks.)

B.1 Explain how the rotation curves of spiral galaxies are determined from observations made at optical wavelengths; comment on how a galaxy's orientation to the line-of-sight affects these measurements. **[6 marks]**

Sketch the form of the rotation curve for a typical spiral galaxy. **[5 marks]**

In the central regions of spiral galaxies, the rotation curve has approximately "rigid-body" form. Explain qualitatively why the central regions of these galaxies cannot *actually* rotate like rigid bodies. **[2 marks]**

Outside the central regions the rotation curve of a particular spiral galaxy is observed to be flat over most of its range, the circular velocity Θ_o being independent of distance r from the centre of the galaxy. Assuming that most of the galaxy's mass is distributed with spherical symmetry, show that the mass $M(r)$ contained within r is given by

$$M(r) = \frac{\Theta_o^{\ 2}}{G} \times r$$ **[6 marks]**

Show that, if the density $\rho(r)$ of matter in this galaxy follows the power-law

$$\rho(r) = \rho_o \left(\frac{r}{r_o} \right)^{\alpha} ,$$

then $M(r)$ is given by

$$M(r) = \frac{4\pi}{(3+\alpha)} \frac{\rho_o}{r_o^{\alpha}} r^{3+\alpha} .$$ **[6 marks]**

Deduce that α has value -2 for this galaxy. **[2 marks]**

Explain briefly why the density must eventually fall off more rapidly than r^{-2}.
[3 marks]

Please turn over the page.

B.2

(a) Give a **qualitative** explanation of epicyclic motion in regions of a galaxy with a constant circular velocity $\Theta(r)$=constant. **[6 marks]**

(b) The epicyclic frequency $\kappa(r)$ at radial distance r from the centre of a spiral galaxy is given in terms of the Oort parameters by

$$\kappa(r) = \sqrt{-4B(r)[A(r) - B(r)]},$$

where the Oort parameters $A(r)$ and $B(r)$ are given in terms of the circular velocity $\Theta(r)$ and its derivative with respect to radius by

$$A(r) = +\frac{1}{2}\left[\frac{\Theta(r)}{r} - \frac{d\Theta(r)}{dr}\right], \quad \text{and} \quad B(r) = -\frac{1}{2}\left[\frac{\Theta(r)}{r} + \frac{d\Theta(r)}{dr}\right].$$

Show that, in a galaxy with a completely *flat* rotation curve, $\kappa(r) = \sqrt{2}\,\Omega(r)$, where $\Omega(r) = \Theta(r)/r$ is the angular velocity of material at radius r in circular orbit about the centre of the galaxy. **[4 marks]**

Discuss whether the orbits of stars undergoing such epicyclic motion will appear closed in an inertial frame. **[6 marks]**

(c) An m-armed spiral density wave in a galactic disc of surface density σ_o obeys the dispersion relation

$$m^2(\Omega_p - \Omega(r))^2 = a^2k^2 + \kappa(r)^2 - 2\pi G\sigma_o\,|\,k\,|$$

where Ω_p is the wave's pattern-frequency, a^2 is the mean-squared turbulent velocity of the gas in the disc, k is the wave's wavenumber, $\kappa(r)$ is the epicyclic frequency, and G is the gravitational constant.

Taking the turbulent velocity a to be zero, show that the range of values of Ω_p required for a wave to propagate are given by

$$\Omega(r) - \frac{\kappa(r)}{m} \le \Omega_p \le \Omega(r) + \frac{\kappa(r)}{m}.$$

[7 marks]

(d) Explain **qualitatively** why resonances occur at

$$\Omega_p = \Omega(r) \pm \frac{\kappa(r)}{m}.$$

and comment on the effect this has on the wave. **[7 marks]**

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B.3 A region of sky of area Ω steradians has been surveyed to find all galaxies with flux densities S greater than a limiting flux density $S_{\lim}$; the distance of each of these galaxies has also been determined. By considering the maximum distance at which a galaxy of luminosity L could still be seen in the survey, show that we expect the survey to contain $N(L)dL$ galaxies in the luminosity range L to $L+dL$, where

$$N(L)dL = \frac{\Omega}{3}\left(4\pi S_{\lim}\right)^{-3/2} L^{3/2}\phi(L)dL,$$

the luminosity function $\phi(L)dL$ being the number of galaxies per unit volume with luminosities in the range L to $L+dL$. **[10 marks]**

From such a survey it is suggested that, over the luminosity range 10^8-10^{11} L_{sun}, the luminosity function has the form

$$\phi(L) = K\left(\frac{L}{L_*}\right)^{-\alpha},$$

Derive expressions for the total number of galaxies per unit volume in the luminosity range 10^8-10^{11} L_{sun} and for the total luminosity of these galaxies. **[8 marks]**

Taking $L_* = 6\times10^9 L_{sun}$, $K = 4\times10^{-13}$ galaxies $\mathrm{Mpc}^{-3}\ \mathrm{L_{sun}}^{-1}$ and $\alpha = 1.4$ show that the total number of galaxies per unit volume in the luminosity range 10^8-10^{11} L_{sun} is ~0.03 Galaxies Mpc^{-3}. **[6 marks]**

If we assumed that the $\phi(L) = K\left(\frac{L}{L_*}\right)^{-\alpha}$ luminosity function was correct for *any* luminosity, we might suppose that we could calculate the total number density and the total luminosity density for galaxies over *all* luminosities. What is the difficulty with doing this? **[4 marks]**

What is the Schecter form of the Luminosity function, and why does it not suffer from that difficulty? **[2 marks]**

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B.4

(a) Discuss **briefly and qualitatively** why gravitational accretion is more likely to power active galactic nuclei than is nuclear fusion. **[5 marks]**

(b) To a good approximation, the kinetic energy T and the potential energy Ω of material in the accretion disc of a black hole satisfy the Virial Theorem:

$$2T + \Omega = 0\,.$$

Show that the energy ΔE radiated by material whose potential energy changes by $\Delta\Omega$ is given by $\Delta E = -\frac{1}{2}\Delta\Omega$. **[6 marks]**

(c) Deduce that the power L radiated by a mass-rate $\dot{m}$ of material spiralling from infinity to distance R from a mass M is given by

$$L = \frac{1}{2}\frac{GM\dot{m}}{R}\,.$$ **[6 marks]**

Remembering that the last stable orbit is at three Schwarzschild radii, and that $R_{Schwarzschild} = \frac{2GM}{c^2}$ use the above result to show that the maximum luminosity $L_{\max}$ of an accretion disc around a non-rotating black hole is given by

$$L_{\max} = \frac{1}{12}\dot{m}c^2\,,$$ **[2 marks]**

independent of the mass of the hole.

(d) Deduce that, if an accretion disc has luminosity L which is variable on a time-scale τ, the mass of the central black hole must satisfy

$$M \le \frac{1}{12}\frac{c^3\tau}{G}$$ **[6 marks]**

(e) The Eddington luminosity limit can be used to derive an upper limit on the mass M. Substitution of numerical values which when combined with the time-scale limit gives:

$$\sim 10^{-31} \times L(\mathrm{W}) \le \frac{M}{M_{Sun}} \le \sim 10^{8} \times \tau(\mathrm{h})\,.$$

Observations show that the luminosities of Active Galactic Nuclei are in the range 10^{33}-10^{40} W, and their variability timescales are in the range of ~10^{-2}-10^{2} hours. In the light of the inequality above and these observations, discuss **briefly** the constraints set by observation on the masses of black holes in Active Galactic Nuclei. **[5 marks]**

End of Examination Paper **Professor J P Emerson**