Brunel University
Queen Mary, University of London
Royal Holloway, University of London
University College London

Intercollegiate post-graduate course in High Energy Physics

Paper 1: The Standard Model

Monday, 28 January 2008

Time allowed for Examination: 3 hours

Answer **ALL** questions

Books and notes may be consulted

The Standard Model

Question 1 (4 marks)

At a collider, two high energy particles, A and B with energies E_A and E_B, which are much greater than their rest masses, collide head on. Derive the expression for the centre-of-mass energy. **[1]**

Using this expression, what would be the centre-of-mass energy of a proposed future facility ("LHeC") which will collide 7 TeV protons with 70 GeV electrons? **[1]**

Now consider particle B (the proton) to be at rest. Derive the formula for the centre-of-mass energy of such a fixed-target experiment. **[1]**

What electron beam energy would be required in the fixed-target experiment in order to achieve the same centre-of-mass energy as in the proposed LHeC facility? **[1]**

Question 2 (12 marks)

Consider the Compton scattering of a photon, $k = (\omega, \vec{k})$, off a stationary electron, $p = (m, \vec{0})$. The photon is scattered through an angle θ and the four momenta of the final state particles are $k' = (\omega', \vec{k'})$ and $p' = (E', \vec{p'})$ for the photon and electron respectively. Derive the Compton shift relation

$$\lambda' - \lambda = 2\lambda_c \sin^2(\theta/2)$$

where $\lambda = 2\pi/\omega$, $\lambda' = 2\pi/\omega'$ and $\lambda_c = 2\pi/m$ **[4]**

Draw the leading order Feynman diagrams for Compton scattering and state whether they are s, t or u channel. **[2]**

Part of the trace calculation for evaluating the cross section involves

$$A = \frac{1}{4} Tr \left[\not{\epsilon}' \not{k} \not{\epsilon} \left(\not{p} + m \right) \not{\epsilon} \not{k} \not{\epsilon}' \left(\not{p}' + m \right) \right],$$

where ϵ and ϵ' are the photon's initial and final polarization four vectors. In a gauge for which $p \cdot \epsilon = p \cdot \epsilon' = 0$, show that

$$A = 2\epsilon^2 k \cdot p \left[2 \left(\epsilon' \cdot k \right)^2 - \epsilon'^{\,2} \left(k' \cdot p \right) \right].$$

Trace theorems for γ matrices need not be derived, but should be quoted. Note that $\not{a}\not{b}\not{a} = 2a \cdot b \not{a} - a^2 \not{b}$. **[6]**

Question 3 (10 marks)

Draw the leading order Feynman diagram for electron-muon scattering, $e^-(k) + \mu^-(p) \rightarrow e^-(k') + \mu^-(p')$, where the four momenta are indicated in the reaction. **[1]**

Simplify the expression for the transition amplitude:

$$|T_{\rm fi}|^2 = \frac{e^4}{4q^4} \mathrm{Tr}\left[(\not{k}' + m)\gamma_\mu(\not{k} + m)\gamma_\nu\right] \cdot \mathrm{Tr}\left[(\not{p}' + M)\gamma^\mu(\not{p} + M)\gamma^\nu\right]$$

such that the Traces are removed, assuming the high-s limit of zero masses. Trace theorems for γ matrices need not be derived, but should be quoted. **[4]**

Use the cross-section definition (in the centre-of-mass system):

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|T_{\rm fi}|^2,$$

where p_f and p_i are the final and initial three-momenta, to derive the cross section.

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta\right),$$

where α is the fine structure constant and θ is the angle between the e^- and μ^-. **[5]**

Question 4 (4 marks)

The branching ratios for $D^+ \rightarrow K_s^0\pi^+$ and $D^+ \rightarrow K^+\pi^0$ are very different, *viz.* 1.47% and $2.37 \cdot 10^{-4}$. Assuming the simple spectator model, draw diagrams for the two decays. **[2]**

Give a reasoning for some of the difference in rate. **[2]**

Question 5 (6 marks)

State what is meant by local and global gauge transformations. **[2]**

From the Lagrangian

$$\frac{1}{8}\left[g_W^2(v+h)^2(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) - (v+h)^2(g'B_\mu - g_W W_\mu^3)(g'B^\mu - g_W W_3^\mu)\right]$$

derive the ZZH and ZZHH couplings. (Simplify your answer to remove any dependency on v.) **[4]**

Question 6 (6 marks)

For $\sqrt{s} = 35$ GeV, what would you expect the value of

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

to be when considering only the EM coupling? At what higher energy would would you expect the value to change? **[3]**

Draw a higher-order diagram (i.e. consideration of the strong force) which would affect this value. **[1]**

Briefly describe how such higher-order diagrams led to the discovery of the gluon. **[2]**

Question 7 (7 marks)

The amplitude for the decay $\pi^-(q) \rightarrow \mu^-(p) + \bar{\nu_\mu}(k)$ is given by:

$$|T_{\text{fi}}|^2 = \frac{G_F^2}{2} f_\pi^2 \cos^2\theta_c m_\mu^2 \text{Tr}\left[(\not{p} + m_\mu)(1-\gamma^5)\not{k}(1+\gamma^5)\right]$$

Use Trace theorems to show this simplifies to

$$|T_{\text{fi}}|^2 = 4G_F^2 f_\pi^2 \cos^2\theta_c m_\mu^2 (p \cdot k)$$

[4]

The ratio of decay rates:

$$R = \frac{\Gamma(K^- \rightarrow e^- + \bar{\nu_e})}{\Gamma(K^- \rightarrow \mu^- + \bar{\nu_\mu})}$$

can be written in terms of the particle masses. Use this relation to give the value to 2 decimal places showing that the rate is close to that measured from experiment, $\sim 2.44 \times 10^{-5}$.
($m_e = 0.511$ MeV, $m_\mu = 105.7$ MeV, $m_K = 493.7$ MeV) **[3]**

Question 8 (12 marks)

Draw the Feynman diagrams of the two leading order (in α_s) processes in deep inelastic ep scattering. **[2]**

The photon emitted from the electron can also be sometimes considered to have a structure, by fluctuating into a pair of quarks. Draw an example Feynman diagram of these so-called "resolved" photon processes. **[2]**

In this way, an ep collider can also be thought of as a hadron collider. Draw all Feynman representations, including the initial hadrons and their products, for the hard scatters, $qq' \to qq'$ and $qq \to qq$. **[4]**

Write down the forms of the (partonic) cross sections for $qq' \to qq'$ and $qq \to qq$ in terms of the Mandelstam variables, s, t and u, associating each term with the relevant Feynman diagram. **[4]**

Question 9 (5 marks)

Contrast the advantages and disadvantages of e^+e^- and pp colliders. Use two headline measurements or major discoveries to justify your answer. **[5]**

Question 10 (6 marks)

What property of the EM interaction means that photons do not self-couple? **[1]**

Draw a Feynman diagram of a process at the LHC in which three gluons couple at one vertex. **[2]**

Explain briefly why the QCD coupling, α_s, has a different behaviour with the scale, Q^2, compared to that of the QED coupling, α. **[3]**

Question 11 (6 marks)

Draw a Feynman diagrams for each of neutral current and charge current deep inelastic scattering at HERA. **[2]**

Draw a sketch of how their cross sections vary with Q^2 and explain the features. **[2]**

The neutral current cross section is sensitive to all quarks in the proton. Which quarks are the charge current cross section for (a) e^+p and (b) e^-p sensitive to? **[2]**