

Experiment C1. Resonance

If you are not familiar with the concept of the impedance of a circuit, you should first read Appendix C in the pink section at the end of these notes. If you still have any difficulties with this concept, discuss it with your tutor.

Objectives

This experiment demonstrates the occurrence of resonance in electrical circuits, specifically *LCR* circuits, and introduces the parameters ω_0 and Q commonly used to characterise resonant phenomena. Both series and parallel resonance are studied, and non-ideal effects, such as the parasitic capacitance of an inductor, are studied.

Pework Questions.

1. What is the impedance of an ideal series LCR circuit?
2. For the circuit in Fig. C1-1, show that the maximum voltage over R is obtained at $\omega = \omega_0$.
3. For an ideal parallel LC circuit, show that the impedance at resonance is infinite. (Hint: see example 6 in appendix C. Think about what this means for the circuit.)

Background

When a capacitor is discharged through an inductor, the current will oscillate at a frequency $\omega = 1/\sqrt{LC}$. Very large oscillations can be induced in such a circuit when it is connected to an external oscillator tuned to this frequency. Such a phenomenon is called a *resonance*. The amplitude of the oscillation, when it is driven at the resonance frequency, is determined by the resistance of the circuit. The smaller the resistance, the bigger the amplitude. Resonance phenomena also occur in mechanically vibrating systems, common examples being a pendulum or spring or quartz crystal driven by an external force at the same frequency as the natural frequency of oscillation.

Consider for example the series *LCR* circuit shown in Fig. C1-1 which is driven by an oscillator of voltage amplitude V_0 and angular frequency ω .

The impedance of this circuit is

$$Z = (R + R_L) + j(\omega L - 1/\omega C)$$

where R_L is the effective loss resistance of the inductor L (due to the wire resistance and power loss in the ferrite core). The current in the circuit is given by $i = v_{in}/Z$ so the voltage, v_R across R is given by

$$v_R = Ri = \frac{Rv_{in}}{(R + R_L) + j(\omega L - 1/\omega C)}$$

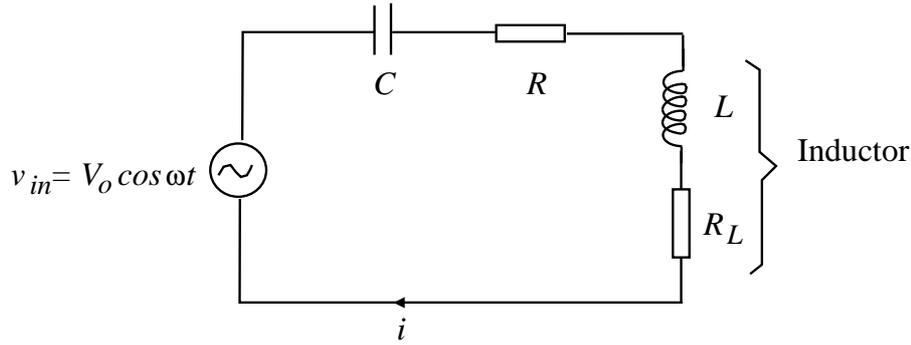


Fig. C1-1 Series resonant circuit

The *amplitude* of v_R is given by

$$|v_R| = V_R = \frac{RV_0}{[(R + R_L)^2 + (\omega L - 1/\omega C)^2]^{1/2}} \quad (1)$$

The phase difference ϕ between v_R and v (or between i and v) is given by

$$\tan \phi = \frac{\omega L - 1/\omega C}{R + R_L} \quad (2)$$

V_R and ϕ are plotted in Fig. C1-2 as a function of ω . At low frequencies i and V_R are small due to the high reactance, $1/\omega C$, of the capacitor. At high frequencies i and V_R are small due to the high reactance, ωL , of the inductor. The impedance of the circuit is a minimum, and i and V_R are a maximum when $\omega L = 1/\omega C$. This condition defines the *resonant frequency* ω_0 given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (3)$$

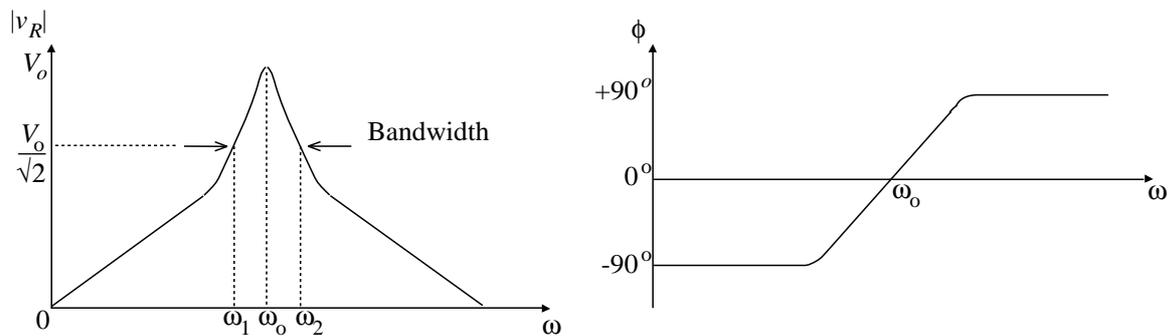


Fig. C1-2 Amplitude and phase response of the circuit in Fig. C1-1

The *bandwidth*, $\Delta\omega$, is defined as the difference between the frequencies ω_1 and ω_2 at which V_R is $1/\sqrt{2} = 0.707$ of its peak value, and is given by

$$\Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q} \quad (4)$$

where Q is the quality factor given by the ratio of inductive reactance at resonance to the total series resistance:

$$Q = \frac{\omega_0 L}{R + R_L} = \frac{1}{R + R_L} \left(\frac{L}{C} \right)^{1/2} \quad (5)$$

Eqn. (4) can be derived by solving the equation $|Z| = \sqrt{2}(R + R_L)$ to find ω_1 and ω_2 . Note that the bandwidth is a measure of the “sharpness” of the resonance — the bandwidth is small if Q is large (or if ω_0 is small). A resonant circuit acts as an energy reservoir and Q measures the ratio of stored energy to the rate of energy dissipation.

Resonance Properties

At resonance, the impedance Z has a minimum value equal to the total series resistance in the circuit i.e. $Z = R + R_L$, and the current through the circuit is a maximum, of amplitude $I_0 = V_0/(R + R_L)$. The impedance of the LC section is $Z = R_L + j(\omega_0 L - 1/\omega_0 C) = R_L$ at resonance, so the voltage across the LC section, v_{LC} , will be $\ll v_R$ if $R_L \ll R$. For an ideal inductor with $R_L = 0$, v_{LC} is also zero due to the exact cancellation of the *equal but opposite-phase* voltages across L and C . In a practical resonant circuit, v_L and v_C are only approximately equal in amplitude, so the cancellation is never perfect.

The voltage across C , for example, is given by $v_C = i/j\omega C$, so at resonance

$$|v_C| = V_C = \frac{I_0}{\omega_0 C} = \omega_0 L I_0 = \frac{\omega_0 L}{R + R_L} V_0 = Q V_0 \tag{6}$$

For a high- Q circuit ($Q \gg 1$), the voltage across C will be *much larger* than the voltage amplitude V_0 of the oscillator.

Procedure

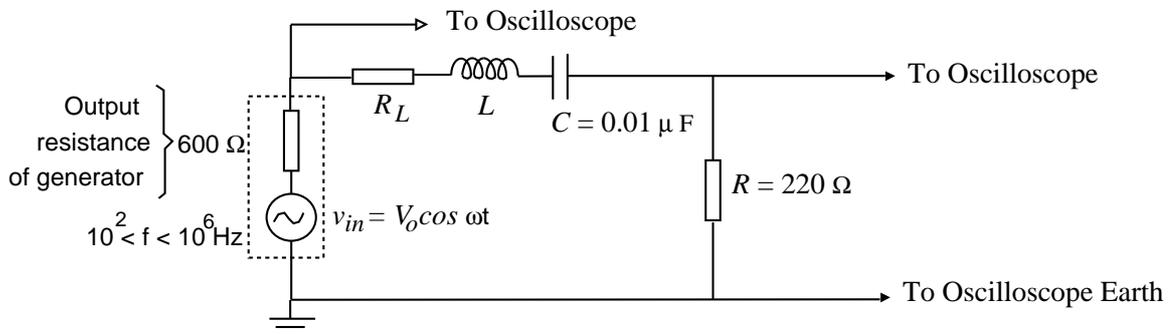


Fig. C1-3 Circuit used to measure frequency response

1. Before connecting the circuit, measure your values of L , R and C on the GR Digibridge at the front of the lab.
2. Connect the series LCR circuit shown in Fig. C1-3 using the external signal generator. The inductor is housed in the cylindrical box available on the benchtop. Note down the identification numbers of your inductor and of the circuit board kit that you are using in your logbook.
3. The loss resistance of the inductor, R_L , is given in the bench notes. The impedance of the LCR circuit varies widely with frequency (down to $\approx 200 \Omega$). As a result, the output resistance of the external signal generator (approximately 600Ω) becomes

important at some frequencies, causing the signal generator voltage, V_0 , to vary with frequency.

Before taking measurements, search for resonance by scanning through 5 powers of 10 in the frequency range. Use the $\times 1$ setting for the oscilloscope probe. The resonance peak will be between 5 and 15 kHz. Note that you can determine what your frequency value is either using the frequency counter on the bench, or using the frequency counter built in to the oscilloscope. Use the tips given below to measure V_R/V_0 vs f over the range $100 \text{ Hz} < f < 1 \text{ MHz}$. Plot a graph of V_R/V_0 vs f using a log-log scale. You will find that your graph needs to cover something like 3 powers of 10 vertically and 4 powers of 10 horizontally. You have the choice to plot your graph on graph paper, or using a computer and QTIplot or Excel. If you use a computer, make sure to save your data so that you can use it again later, as you will need to in the next section.

Tips:

- Do a quick run through the full frequency range to begin with, plotting, say, three points per decade — use finer intervals later, where necessary.
- Note down any frequencies at which any unusual behaviour is observed.
- It is essential to plot points as you go.
- You are determining ratios of voltages. It is actually better technique, as you may have been shown in first year laboratory, to measure the ‘peak to peak’ voltages V_{pp} rather than the ‘maximum voltages’ (amplitudes) V_{max} , of the signals v_R and v_0 .
- Since you are only interested in the ratio V_R/V_0 , choose a value of V_0 which allows you to measure V_R accurately. For frequencies close to resonance it is recommended that you keep $V_0 \leq 1 \text{ V}$ to avoid core saturation (and prevent annoying whistles from the inductor). Away from resonance you can increase the input voltage to maximum.

C1 ▷

4. At low frequencies the term $1/\omega C$ dominates in eqn. (1) so $V_R \approx \omega C R V_0$. At frequencies well above resonance you would expect that $V_R \approx R V_0 / \omega L$.

Using these approximations, plot on your graph the theoretical response in the intervals 100 Hz – 1 kHz and 20 kHz – 1 MHz and comment on the agreement (or otherwise). We will return to this point in the question at the end of the experiment.

5. Using measured values of L and C , compare (with measurement uncertainties taken into account) the theoretical and experimental values of
- (a) ω_0 ; the frequency dial of the generator is only accurate to about $\pm 5\%$, so the resonant frequency should be measured carefully with the frequency counter on your bench, using the fact that $\phi = 0$ at resonance (eqn. 2). Carry out this measurement several times in order to estimate uncertainties.
 - (b) V_R/V_0 at $\omega = \omega_0$; this should be a separate measurement, not simply read from the graph.

C2 ▷

- Interchange R and C so that you can measure V_0 and V_C at resonance; the frequency must be set carefully to the point where V_0 is a minimum. Draw the modified circuit. Use this result and eqn. (6) to obtain an experimental value for Q , and compare with the theoretical value given by eqn. (5).

C3 ▷

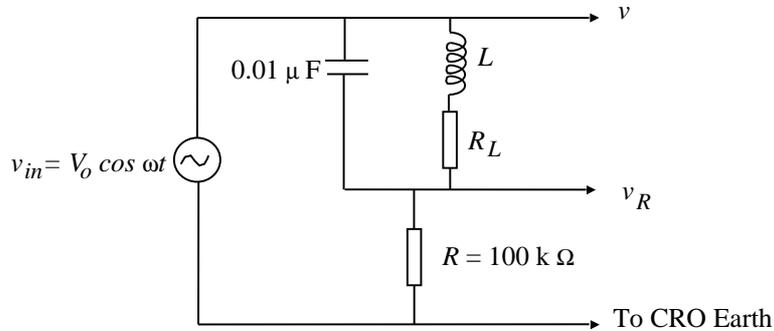


Fig. C1-4 Parallel LCR circuit

- The impedance of a *parallel LC* circuit is infinite for ideal inductors and capacitors at the resonant frequency $\omega_0 = 1/\sqrt{LC}$. If the finite resistance, R_L , of the inductor is taken into account, the impedance is given by $Z_0 = L/CR_L$ (see Example 6 in the appendix). Check these predictions experimentally using the circuit in Fig. C1-4. At resonance the phase shift between v_R and v is zero and $V_R = RV_0/(R + Z_0)$ where $R = 100\text{ k}\Omega$. Use a $\times 10$ probe to measure v_R .

Question: Previously, you will have noticed a substantial discrepancy at high frequency between your measurements and the theoretical prediction. You can assume that this discrepancy is caused by the presence of an “additional” component not shown on the circuit diagram. What *is* the additional component, where does it come from and what is its value? Does it explain the slope of your graph near 1 MHz?

Conclusion

Summarize the important results of this experiment. In particular, comment on the importance when constructing a practical electronic circuit of accounting for the non-ideal behaviour of the components used.

C4 ▷