UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Physics 2B29: Electromagnetic Theory

COURSE CODE	:	**PHYS2B29**
UNIT VALUE	:	**0.50**
DATE	:	**08-MAY-02**
TIME	:	**10.00**
TIME ALLOWED	:	**2 hours 30 minutes**

02-C1081-3-140

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Answer SIX questions from SECTION A and THREE from SECTION B.
The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may need:

Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}.

The Poynting vector averaged over a whole number of cycles of a plane E.M. wave is $\langle \mathbf{N} \rangle = 1/2\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2$,where E_0 is the magnitude of the electric vector.

The solid angle subtended by a sphere is 4π steradians.

$$\nabla V(\mathbf{r}) = \hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial V}{\partial \theta} + \hat{a}_\phi \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

The speed of light *in vacuo* is $c = 3 \times 10^8$ ms^{-1}.

SECTION A

1. Write down the integral form for Ampere's law in its generalised form and briefly state what it means. [3]

   Show how it leads to the Maxwell equation $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$. [4]

2. Write down the relationship between the electric field strength **E**, the electric dispacement **D** and the polarisation **P**. State the units for each quantity. [3]

   Briefly explain the difference between free charge and polarisation charge. [2]

   Give a defining equation for relative permittivity ε_r. What are its dimensions? [2]

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3. Use appropriate defining equations to derive the dimensions of the following electromagnetic quantities in terms of the basic dimensions of mass $[M]$, length $[L]$, time $[T]$ and charge $[Q]$:-

   electric field strength **E**, magnetic induction **B**, electric polarisation **P**,
   electric induction **D**, magnetic field strength **H**, magnetisation **M**,
   magnetic suceptibility χ_m, electric susceptibility χ_e,
   permeability of free space μ_0, permittivity of free space ε_0. [7]

4. Show that the tangential component of **E** and the normal component of **D** are continuous across a plane boundary between different media at which there are no surface charges. [7]

5. A small electrical dipole of strength **m**, pointing along the z-axis at the origin of a polar coordinate system with θ measured from the z-axis, gives an electrostatic potential at position **r**

$$V(\mathbf{r}) = \frac{m\cos\theta}{4\pi\varepsilon_0 r^2}.$$

   Obtain expressions for the three components of the field $\mathbf{E}(\mathbf{r})$. [2]
   Sketch the shape of the field lines when $|\mathbf{r}|$ is much greater than the size of the dipole. [2]
   At what angles θ is **E** at 45° to **r**? [3]

6. Sketch the hysteresis loop for a typical ferromagnetic material. Label the two axes and mark the point corresponding to the onset of saturation. Also mark the points at which coercivity H_c and remanence B_r are defined. State briefly what saturation, coercivity and remanence mean. [3]

   Give a typical value for the saturation magnetic induction B_s for the steel from which electromagnets are made. [2]

   Why it is impossible to give a unique number for the relative permeability of a ferromagnetic material? [2]

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7. Explain briefly, with the aid of appropriate sketches, what happens to the oscillations of the **E** vector in the following kinds of light:

plane polarised,
elliptically polarised,
circularly polarised,
unpolarised,
partially polarised. [7]

8. What are the units of the Poynting vector **N**? [1]

Assuming a plane wave is made up from photons with energy $\hbar\omega$, obtain an expression for the flux of photons through unit area, in terms of E_0 and ω. Deduce from this how much momentum the wave carries per second. [3]

A perfectly absorbing black carbon granule is emitted from a comet approaching close to the Sun. Work out the force on the granule due to radiation pressure when the rate of incoming solar heat is 1 milliwatt. [3]

SECTION B

9. A long permanent magnet may have the same values for the internal and external magnetic induction field **B(r)** as those for a solenoid of the same dimensions. Explain how the **H** and **M** fields differ between the two cases. [4]

Using the differential form of the Ampere law, and explaining any assumptions or approximations made, show that the magnetic induction **B** inside a long cylindrical solenoid is uniform at points well away from the ends. Derive an expression giving the size of **B** inside a long solenoid in terms of N, the number of turns per metre, and the current I. [6]

A long cylindrical solenoid and a long bar magnet, with the same cross sectional areas and the same uniform internal **B** field are placed end to end with axes collinear. Sketch the lines of **B** and **H** both inside and outside the two systems in the region where they meet. [4]

If the solenoid has 1000 turns per metre, carrying 10 Amperes, what must be the size of **M** in the matching bar magnet? [6]

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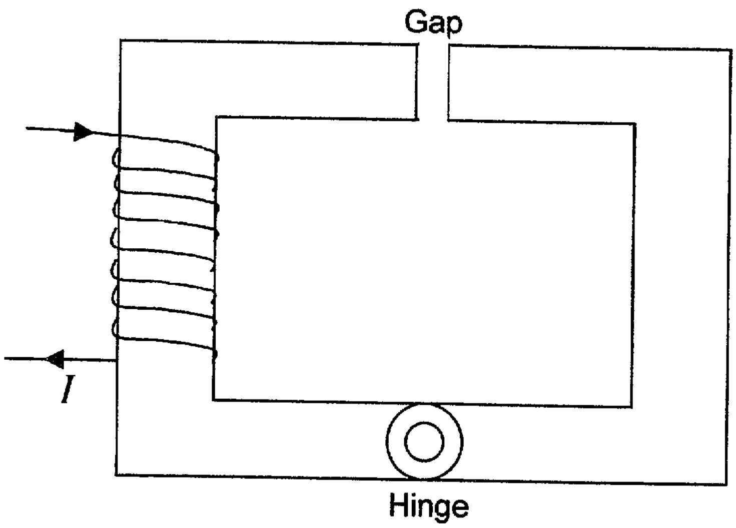
10. In a system of fixed rigid circuits and pieces of linearly magnetisable materials the stored magnetic energy can be written as $U_m = \frac{1}{2}\sum_i I_i \Phi_i$, where I_i is the current flowing in the ith circuit and Φ_i is the magnetic flux through the ith circuit. Show by considering a small displacement of an element of the system, with all currents maintained at steady values, that the force on that element in the direction of its displacement is equal to the rate of change of stored magnetic energy per unit displacement. [6]



The sketch shows a simple magnetic clamp which has a frictionless hinge in its ferromagnetic yoke, constructed so that the magnetic flux through the hinge is not interrupted as it bends through small angles. A coil of N turns is wound on the yoke and carries current I. With the simplifying assumptions that the cross section of the yoke A is constant around the whole length l of its circumference, and that the yoke can be treated as a linear magnetic material with well defined relative permeability μ_r, obtain an expression for the force which the clamp exerts on a sample of non-magnetisable material of thickness s placed in the gap. Comment briefly upon the validity of the two assumptions. [9]

If the gap has been allowed to close so that there is no discontinuity in the yoke, calculate the force required to begin to open it again. Take N=500 turns, I = 1 A, l = 0.15m, A = 0.0001 m^2, μ_r= 1500. [5]

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11. The Fresnel relations for light reflected and refracted at a plane uniform dielectric surface can be written as

$$t_\perp = \frac{2\cos\alpha}{\cos\alpha + (n_2/n_1)\cos\alpha'}$$

$$r_\perp = \frac{\cos\alpha - (n_2/n_1)\cos\alpha'}{\cos\alpha + (n_2/n_1)\cos\alpha'}$$

$$t_\parallel = \frac{2\cos\alpha}{(n_2/n_1)\cos\alpha + \cos\alpha'}$$

$$r_\parallel = \frac{(n_2/n_1)\cos\alpha - \cos\alpha'}{(n_2/n_1)\cos\alpha + \cos\alpha'}.$$

Draw a ray diagram and use it to define all of the quantities in these relations. [3]

What special behaviour occurs at the Brewster angle α_B? Use the appropriate relation, together with Snell's law, to confirm that $\alpha_B = \tan^{-1} n_2/n_1$. [6]

Show that there can be a Brewster angle both when $n_1 < n_2$ and when $n_1 > n_2$. [3]

Unpolarised sunlight is reflected from a clean and polished road surface with refractive index 1.6. What is the Brewster angle for this surface? A driver sees a reflection of the sun in the road surface at an angle of reflection equal to 45°. Calculate the factor by which the energy carried by this reflected light to his eyes is reduced when he puts on a pair of "perfect" polarising sunglasses with the optimal alignment. ("Perfect" implies complete transmission of light with one polarisation plane and complete absorption with the orthogonal plane). [8]

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12. Given the equation $\nabla^2\mathbf{E} - \mu\sigma\frac{\partial\mathbf{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$, show that plane electromagnetic waves in a uniform conducting medium obey the dispersion relation $k^2 = \mu\varepsilon\omega^2\left(1 + {}^{i\sigma}\!/_{\varepsilon\omega}\right)$. Define all the symbols in these two equations. What is the phase velocity of such a wave for $\sigma << \varepsilon\omega$? [5]

Show that the amplitude of waves penetrating normally to a distance d in a strongly conducting medium with $\sigma >> \varepsilon\omega$ are damped by a factor $e^{-d/\delta}$ where the skin-depth $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$. [5]

A DC electrical signal and radio frequency signals of 1 MHz and 100 MHz are transmitted in turn through a single aluminium alloy conductor 1 km long with radius 1mm and conductivity $\sigma = 4\times10^7$ (ohm metres)$^{-1}$. There are no other conductors anywhere near it. What is its resistance for each of the signals, under the approximation that conduction is uniform over a layer one skin-depth thick on the outside of the conductor? [You may assume $\mu_r = 1$ for aluminium.] [10]

13. Sketch the instantaneous shape of the lines of electric field **E** in the plane containing a small Hertzian dipole of length ℓ in vacuum which is oscillating with angular frequency ω and maximum current I_0. You should show the lines at a moment when all of the charge is concentrated at the ends of the dipole. The sketch should go out to a radius r ~L from the dipole, where $L = 2\pi(c/\omega)$. Explain with reference to your sketch what is meant by the retarded time. [7]

At distances very much greater than L the instantaneous electric and magnetic fields in the outgoing waves due to such a dipole are given in spherical polar coordinates by

$$E_\theta = \frac{j\omega\ell I_0 \sin\theta}{4\pi\varepsilon_0 c^2} \cdot \frac{\exp\{j(\omega t - kr)\}}{r}$$

and $$B_\phi = \frac{j\omega\ell I_0 \sin\theta}{4\pi\varepsilon_0 c^2} \cdot \frac{\exp\{j(\omega t - kr)\}}{cr}.$$

Explain briefly why there are no other significant components to either field. Use these expressions to show that the outflow of power in waves from the dipole per unit solid angle, averaged over a large number of cycles and over all directions, is given by the expression $\overline{W} = \frac{\omega^2 I_0^2 \ell^2}{48\pi^2\varepsilon_0 c^3}$ watts per steradian.

What is the ratio of this outflow of power to the outflow per unit solid angle in the equatorial plane, perpendicular to the dipole moment? [9]

For a given value of I_0 the power emitted by the dipole apparently grows as ℓ^2. Discuss why this rate of growth only applies for $\ell << L/2$. [4]

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